EE 311 **January 9, 2016**

Chapter 1 problems

1. What is the difference between the transient response and the sinusoidal steady state response of a digital filter.

5. FIR filters do not have a feedback path. What are the implications of this for system stability? Is oscillation possible? Why or why not?

6. IIR filters have a feedback path and FIR filters do not. What would you expect to be the result of small errors in the formulation of the coefficients for these two systems? Would the impact of such an error be greater or less for an IIR filter? Explain.

8. Since it is often difficult in practice to approximate an impulse, describe another method of finding the impulse response indirectly from say, a step function which is readily available on most signal generators.

12. Suppose the impulse response function for an FIR difference equation is symmetric. For example the response might be given by
$h(nT) = \{b_0, b_1, b_2, b_3, b_4, b_5, b_4, b_3, b_2, b_1, b_0\}$.
How can the difference equation be written to use this symmetry to reduce the number of multiplications necessary for its evaluation?

Hint

The difference equation can be written as:

$$y(n) = b_0 x(n) + b_1 x(n-1) + ... + b_4 x(n-4) + b_5 x(n-5) + b_4 x(n-6) + ... + b_0 x(n-10)$$

Which can be written as:

$$y(n) = b_0[x(n) + x(n-10)] + b_1[x(1) + x(n-9)] + ... + b_4[x(n-4) + x(n-6)] + b_5 x(n-5)$$

Analysis and Design Problems

1.1 Using the numerical integration by trapezoids outlined in this chapter find the difference equation for a digital filter to approximate the analog filter below.

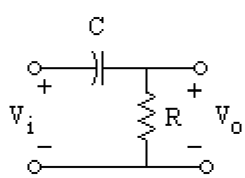


Figure P1.1

An analog filter.

Answer

$$v_0(n) = K_1[v_i(n) - v_i(n-1)] + K_2 v_o(n-1)$$

Where $K_1 = \frac{2RC}{2RC+T}$ and $K_2 = \frac{2RC-T}{2RC+T}$

1.2 A typical computer program for a digital filter was given as:

```
Initialize Variables
DO Forever
   Call AtoD(Vi)       ;Get a sample from the A to D
   Vo = K1*Vi + K2*Vo1
   Call DtoA(Vo)       ;Output Vo to D to A
   Vo1 = Vo            ;Reset the value of the old variable.
   Wait for T seconds to pass
Loop
End
```

Answer the following questions about the program:

A) What is the purpose of the statement "Wait for T seconds to pass." What are the consequences of removing this statement.

B) What variable(s) need to be initialized by the "Initialize Variables" statement.

C) If the A to D converter takes 10 microseconds to complete a conversion, the computer requires 8 microseconds for each multiply and .5 microseconds for each add and the D to A converter requires .1 microsecond, approximately what is the maximum sample frequency for this filter with this difference equation.

Answer

B) K1 and Vo1

C) There are 2 multiplies and 1 add = 16.5 μsec plus the A/D = 8 μsec plus the D/A = 0.1 μsec comes to 24.6 μsec. Maximum sample frequency s 1/24.6μsec = 40.65 KHz.

1.3 For the difference equation given by:

$$v_o(nT) = 0.3333v_i(nT) + 0.3333v_i(n-1)T + 0.3333v_o(n-1)T$$

The sample frequency is 1 KHz.

A) Find and tabulate the impulse response.

B) Find and tabulate the step response.

C) Express the frequency response of the output/input in terms of sines and cosines.

D) Plot the magnitude and phase of the frequency response for the frequency range 0 to fs/2.

E) Does this filter represent a FIR or an IIR filter? Explain.

Answer

A)

n	**v_i**	**$v_i/3$**	**$v_i(n-1)/3$**	**$v_o(n-1)/3$**	**v_o**
0	1	1/3	0	0	1/3
1	0	0	1/3	1/9	4/9
2	0	0	0	4/27	4/27
3	0	0	0	4/81	4/81
4	0	0	0	4/243	4/243
…	…	…	…	…	…

B)

n	**v_i**	**$v_i/3$**	**$v_i(n-1)/3$**	**$v_o(n-1)/3$**	**v_o**
0	1	1/3	0	0	1/3
1	1	1/3	1/3	1/9	7/9
2	1	1/3	1/3	7/27	25/27

3	1	1/3	1/3	25/81	79/81
4	1	1/3	1/3	79/243	241/243
…	…	…	…	…	…

C) The difference equation is

$$v_o(nT) = 0.3333 v_i(nT) + 0.3333 v_i(n-1)T + 0.3333 v_o(n-1)T$$

Let $v_i(nT) = e^{j\omega nT}$, $v_i(n-1)T = e^{j\omega(n-1)T}$, $v_o(nT) = Ae^{j\omega nT}$ and $v_o(n-1)T = Ae^{j\omega(n-1)T}$

This gives

$$Ae^{j\omega nT} = (1/3)e^{j\omega nT} + (1/3)e^{j\omega(n-1)T} + (1/3)Ae^{j\omega(n-1)T}$$

Divide both sides by $e^{j\omega nT}$

$$A = (1/3) + (1/3)e^{-j\omega T} + (1/3)Ae^{-j\omega T}$$

Solve this for A to get

$$A = \frac{(1/3)(1+e^{-j\omega T})}{1-e^{-j\omega T}}$$

$$\frac{v_o(e^{j\omega nT})}{v_i(e^{j\omega nT})} = \frac{Ae^{j\omega nT}}{e^{j\omega nT}} = A$$

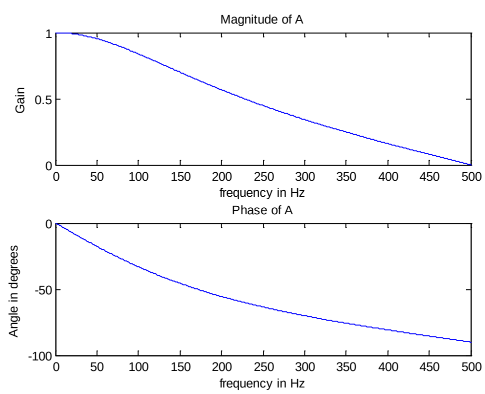
Applying Euler's identity we get

$$\frac{v_o(e^{j\omega nT})}{v_i(e^{j\omega nT})} = \frac{(1/3)[\cos(\omega T) + 1 - j\sin(\omega T)]}{1-\cos(\omega T) + j\sin(\omega T)}$$

$$\left|\frac{v_o e^{j\omega nT}}{v_i e^{j\omega nT}}\right| = (1/3)\frac{\sqrt{(\cos(\omega T)+1)^2 + \sin^2(\omega T)}}{\sqrt{(1-\cos(\omega T))^2 + \sin^2(\omega T)}}$$

$$\theta(\omega T) = \tan^{-1}\left(\frac{\sin(\omega T)}{\cos(\omega T)+1}\right) - \tan^{-1}\left(\frac{\sin(\omega T)}{1-\cos(\omega T)}\right)$$

D) Using MATLAB®



E) This represents an IIR filter since it has feedback.

1.7 Find the general expression for the impulse response for the FIR filter shown in Figure P1.7. Show that this response is always finite in length and has a length no greater than n+1 where n is the subscript of the final coefficient.

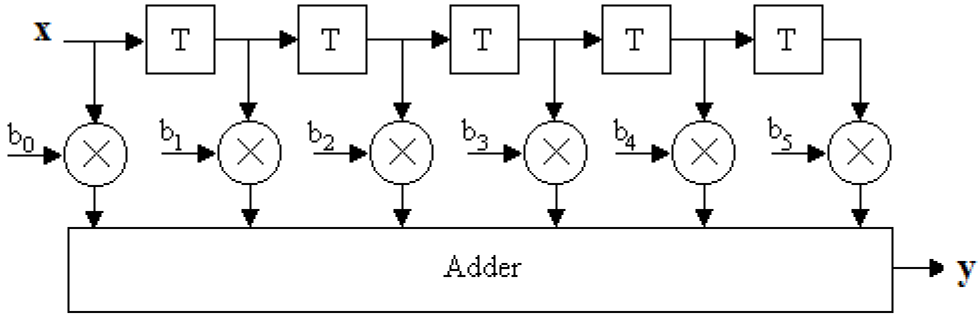


Figure P1.7
An FIR filter.
Answer
$h(nT) = \{b_0, b_1, b_2, b_3, b_4, b_5, 0, 0, 0, \ldots\}$

1.8 Answer the questions below for the difference equation given by:
$v_o(kT) = 0.5v_i(kT) - .035v_i(k-1)T - 0.825v_0(k-1)T$

A) Does this equation represent an IIR or an FIR system? Explain.
B) Write the first 5 terms of the impulse response for this system.
C) Write the expression for the frequency response for this system if the sample frequency is 1,000 Hz.

Solution
A) This is an IIR system since it has feedback.
B) h(k) = {0.5, -0.4475, 0.3692, -0.3046, 0.2513 …}
C) The difference equation is:

The magnitude response is:

$$\left|\frac{v_o e^{j\omega nT}}{v_i e^{j\omega nT}}\right| = (1/3)\frac{\sqrt{(0.5 - 0.035\cos(\omega T))^2 + 0.035^2\sin^2(\omega T)}}{\sqrt{(1 - 0.825\cos(\omega T)^2 + 0.825^2\sin^2(\omega T)}}$$

The phase response is:

$$\theta(\omega T) = \tan^{-1}\left(\frac{0.035\sin(\omega T)}{0.5 - 0.035\cos(\omega T)}\right) - \tan^{-1}\left(\frac{0.825\sin(\omega T)}{1 - 0.825\cos(\omega T)}\right)$$

T = 0.001.

1.9 Consider the difference equation given by
$y(nT) = x(nT) + Ky(n-1)T$

A) Find the expression for the frequency response for y/x in terms of K. Take T = 1.
B) Use MATLAB® to plot the frequency response for two cases: K = 0.8 and K = 1/0.8 = 1.25. What is the same and what is different about these two responses?
C) Use MATLAB® to plot the impulse response for two cases: K = 0.8 and K = 1/0.8 = 1.25. Plot at least 10 terms.
D) What can you conclude from the impulse response of these two difference equations that is not evident from the frequency response?

Answer

A)

The magnitude response is:

$$\left|\frac{ye^{j\omega nT}}{xe^{j\omega nT}}\right| = \frac{1}{\sqrt{(1-K\cos(\omega T)^2 + K^2\sin^2(\omega T)}}$$

The phase response is:

$$\theta(\omega T) = 0 - \tan^{-1}\left(\frac{K\sin(\omega T)}{1-K\cos(\omega T)}\right)$$

B) The magnitude plots have the same shape but when K = 0.8 the magnitude is overall slightly higher. If both plots were normalized to a gain of unity at 0 Hz they would be the same. The phase plots are very different. For K = 0.8 the phase curve begins and ends at 0^o. It reaches a maximum of about -50^o at about 0.1 Hz. When K = 1.25 the phase curve begins at -180^o and ends at 0^o. The total phase changes is about 180^o.

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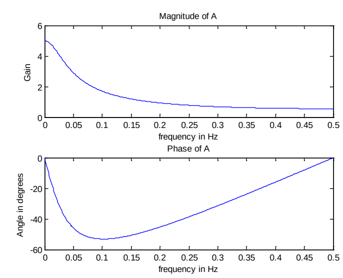
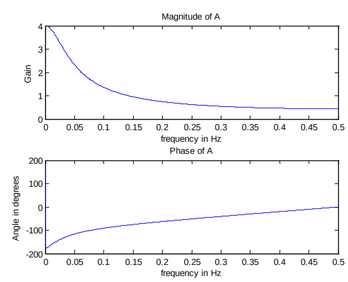
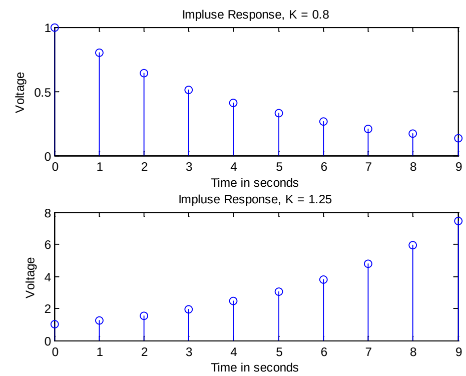




C)



D) When K = 1.25 the system is unstable.

1.14 An N-stage delay line is shown in Figure P1.15.
A) Write the difference equation for the output.
B) Write the frequency response for y/x.
C) Show that the magnitude of the frequency response is always 1 and the phase shift is linearly related to the frequency.

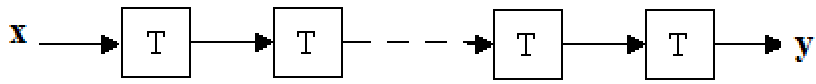


Figure P1.14
A delay line

Solution

A) $y(n) = x(n-N)$

B)

$A = e^{-j\omega NT}$

C) Magnitude of A

$|A| = 1$

Phase of A

$\angle A = -\omega NT$