EE 311 **January 18, 2017**
Chapter 3 Problems

9. What is the relationship between the DFT and the FFT?

11. What is the relationship between the Fourier transform and the Laplace transform?

12. What is the relationship between the Laplace transform and the *z* transform?

13. What is the relationship between the DTFT and the *z* transform?

3.2 Derive the complex-exponential Fourier series coefficients for the periodic signal shown in Figure P3.2.

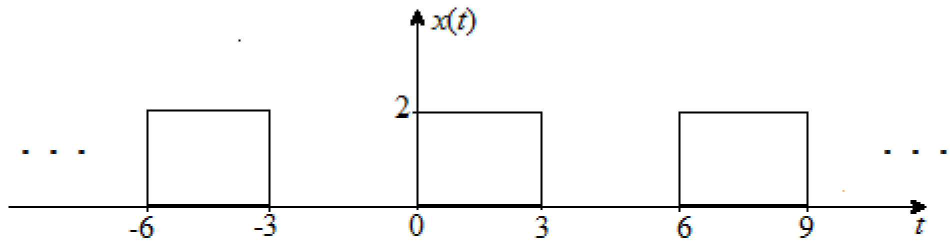


Figure P3.2

Solution

$$C_n = \frac{1}{6}\int_0^3 2e^{-jn\pi/3}dt = \frac{1}{-jn\pi}\left(e^{-jn\pi}-1\right) = \frac{1}{n\pi/2}\left(\frac{e^{jn\pi/2}-e^{-jn\pi/2}}{2j}\right)e^{-jn\pi/2}$$

$$C_n = \text{sinc}\left(n\pi/2\right)e^{-jn\pi/2}$$

3.7. Use MATLAB® and the Fourier series to calculate and plot a 10 Hz square wave with a magnitude of 3 volts. Compute and plot at least one full cycle of the square wave using each specified number of terms (N) of the Fourier series: (a) N = 3, (b) N = 9 , (c) N = 33. (Note: the MATLAB® function *fouriersq* listed in Appendix D.1 can facilitate the solution to this problem.)

Solution

```
fs=5000;
pts=1000;
terms=3;
f0=10
y=3*fouriersq(fs,pts,f0,terms);
y(1001)=0
dt=1/fs;
t=(0:dt:pts*dt);
plot(t,y), grid
```

3.12. (a) Determine the Fourier transform transfer function (frequency response function) of the electrical network shown in Figure P3.12 (a).
(b) Repeat parts (a) for the network shown in Figure P3.12 (b).
(c) Find the impulse response functions for the networks shown in P3.12 (a) and (b).

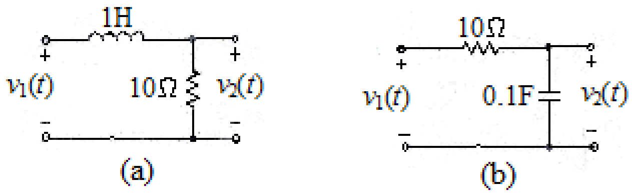


Figure P3.12

Solution
(a)

$$H_a(\omega) = \frac{R}{R + j\omega L} = \frac{R/L}{R/L + j\omega} = \frac{10}{10 + j\omega} = \frac{1}{1 + j\,\omega/10}$$

(b)

$$H_b(\omega) = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\omega}$$

(c)

From Table 3.3: $e^{-at}u(t), \mathrm{Re}\{a\} > 0 \overset{F}{\leftrightarrow} \dfrac{1}{a + j\omega}$

$$h_a(t) = 10e^{-10t}u(t); \quad h_b(t) = e^{-t}u(t)$$

Section 3.2 Problems

3.24. The signal $x(t) = u(t) - u(t-4)$ is shown in Figure P3.24.

(a) Compute the four-point DFT of the signal when it is sampled with $T_s = 2$ ms beginning at $t = 0$. Plot the magnitude and phase spectra.

(b) Use MATLAB® to compute and plot the eight-point DFT of the signal when it is sampled with $T_s = 1$ ms.

(c) Use MATLAB® to compute and plot the sixteen-point DFT of the signal when it is sampled with $T_s = 0.5$ ms.

(d) Compare the results of parts (a), (b), and (c). Comment on their relationships.

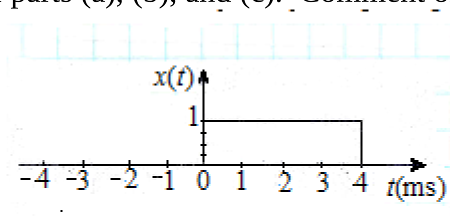


Figure P3.24

Solution

(a)

$$x[n]=\begin{cases}1, & n=0,1,2\\ 0, & otherwise\end{cases}; \quad X[k]=\sum_{n=0}^{3}x[n]e^{-\frac{j2\pi kn}{4}}, k=0,1,2,3$$

$$X[0]=1+1+1+0=3$$

$$X[1]=1+1e^{-\frac{j2\pi}{4}}+1e^{-\frac{j4\pi}{4}}+0e^{-\frac{j6\pi}{4}}=1+e^{-\frac{j\pi}{2}}-1+0=e^{-\frac{j\pi}{2}}=-j1$$

$$X[2]=1+e^{-\frac{j4\pi}{4}}+e^{-\frac{j8\pi}{4}}+0+0=1-1+1=1$$

$$X[3]=1+e^{-\frac{j6\pi}{4}}+e^{-\frac{j12\pi}{4}}+0+0=1+j-1=j1$$

$$\Rightarrow X[k]=[1,-j,\ 1,\ \ j\ ]$$

(b)

```
>> x=[ 1 1 1 1 1  0 0 0 ];
>> X=fft(x);
>> figure(1), stem(abs(X))
>> figure(2), stem(angle(X))
```

(a)

```
>>  x=[ 1 1 1 1 1  1 1 1 1 0 0 0 0 0 0 0 ];
>> X=fft(x,16);
>> figure(1), stem(abs(X))
>> figure(2), stem(angle(X))
```

3.25. (a) Compute the eight-point DFT of the signal shown in Figure P3.20(a) .
(b) Use MATLAB® to confirm the results of Part (a).

Solution

(a)

$$X[k]=\sum_{n=0}^{7}x[n]e^{\frac{-j2\pi kn}{8}}=\sum_{n=0}^{7}(0.5)^n e^{\frac{-j\pi kn}{4}}, k=0,1,2,\cdots,7$$

$$X[k]=1+0.5e^{-\frac{j\pi k}{4}}+0.25e^{-\frac{j\pi k}{2}}+0.125e^{-\frac{j3\pi k}{4}}+0.0625e^{-jk\pi}$$

$$+0.03125e^{-\frac{j5\pi k}{4}}+0.015625e^{-\frac{j3\pi k}{2}}+0.0078125e^{-\frac{j7\pi k}{4}},\quad k=0,1,2,\cdots,7$$

$X[0]=1.9922$; $X[1]=1.1861-j0.6487$; $X[2]=0.7969-j0.3984$; $X[3]=0.6889-j0.1799$;
$X[4]=0.6641$; $X[5]=0.6889+j0.1799$; $X[6]=0.7969+j0.3984$; $X[7]=1.1861+j0.6487$

(b)

```
>> for n=0:7
      x(n+1)=(0.5)^n;
   end
>> X=fft(x,8)
```

3.29. (a) Use MATLAB® to compute and plot the sixty-four point DFT of the signal shown in Figure 3.20(c). Assume a sampling period of T = 0.1 s. Use zero padding if it is required.
(b) What is the frequency resolution of the DFT?

Solution
(a)

```
>> v=zeros(1,64);
>> for n=1:5
       v(n)=2;
   end
>> V=fft(v,64)
```

(b)

$$\Delta\Omega=\frac{2\pi}{N}=\frac{\pi}{32}=0.03125\pi \text{ (rad)};\qquad \Delta\omega=\frac{2\pi}{NT}=\frac{2\pi}{(64)(0.1)}=0.3125\pi \text{ (rad/s)}$$

3.34. (a) Use the four-point FFT signal flow diagram to solve for the DFT of the sequence shown in Figure P3.34.
(b) Use MATLAB to confirm the results of part (a).

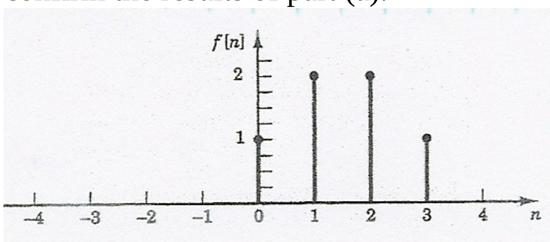
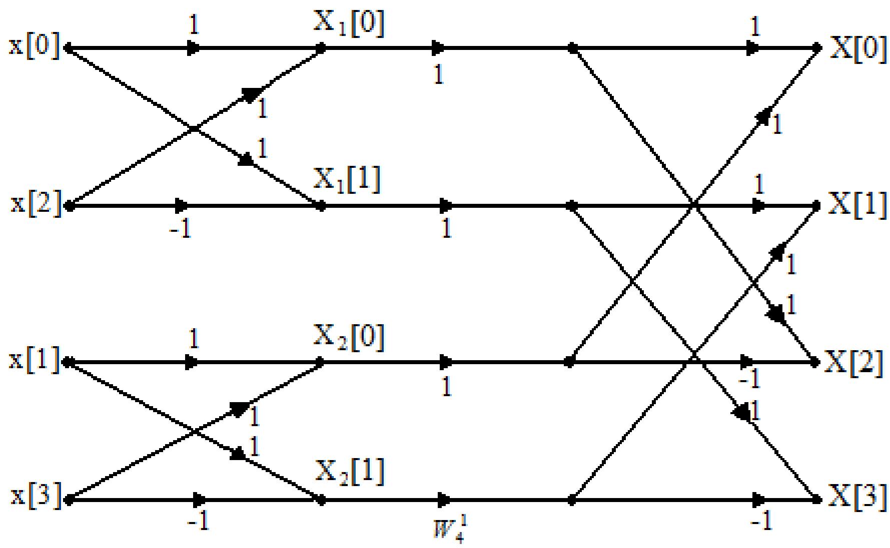


Figure P3.34

Solution
(a) From Figure P3.34: $f[n]=[1,2,2,1]$. $W_4^1=e^{-j\pi/2}=-j$.



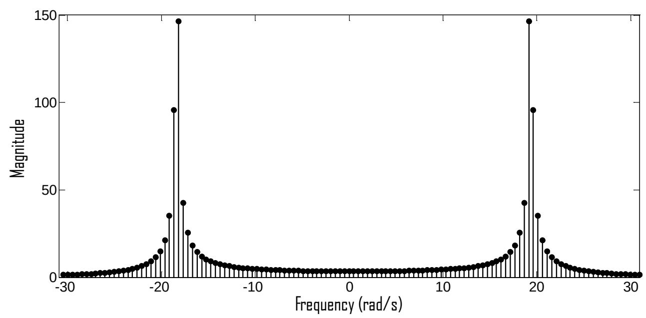
From the diagram:
X[0] = 1 + 2 + 2 + 1 = 6
X[1] = 1 – 2 + (2 – 1)(-j) = -1 – j1
X[2] = 1 +2 – (2 + 1) = 0
X[3] = 1 – 2 – (- 1 + 2)(-j) = -1 + j1

```
(b) >> f = [1 2 2 1];
    >> F = fft(f,4);
```

3.38. The signal $x(t) = 3\cos(6\pi t)$ is sampled 128 times starting at $t = 0$ using a sampling period of 0.1 second.
(a) Use MATLAB® to compute the FFT of the sample sequence.

Solution
(a)

```
>> t=[0:0.1:12.7];
>> x=3*cos(6*pi*t);
>> X=fft(x,128)
>>XM=fftshift(MX);
>> stem(w,XM)
```



3.47. Give the discrete-time function

$x[n] = a^n u[n] - b^{2n} u[-n-1]$,

(a) determine the conditions on *a* and *b* for the z-transform to exist.

(b) assume that the z-transform exists. Find the z-transform and its region of convergence.

Solution

Refer to equations (3.68) and (3.71).

(a) $a^n u[n] \leftrightarrow \frac{z}{z-a}; \quad |z| > |a.|; \quad -b^{2n} u[-n-1] \leftrightarrow \frac{z}{z-b^2}; \quad |b^{-2} z| < 1 \Rightarrow |z| < |b^2|$

(b) $X(z) = \frac{z}{z-a} + \frac{z}{z-b^2}, \quad |a| < |z| < |b^2|$

3.48. Find the z-transforms and the regions of convergence for the following functions:

$(a)\ 0.9^n u[n]$

$(b)\ 0.9^n u[n+1]$

$(c)\ 0.9^n u[n-2]$

$(d)\ 0.9^n u[-n-1]$

$(e)\ 0.9^{-n} u[n+1]$

$(f)\ 0.9^n u[-n]$

Solution

(a)

$$0.9^n u[n] \leftrightarrow \frac{z}{z-0.9}, \quad |z| < 0.9$$

(b)

$$0.9u[n+1] = 0.9^{-1}\left[0.9^{n+1} u[n+1]\right] \leftrightarrow 0.9^{-1}\left[\frac{z^2}{(z-0.9)}\right] = \frac{1.111z^2}{z-0.9}, \quad |z| > 0.9$$

(c) $$0.9^n u[n-2] = 0.9^2\left[0.9^{n-2} u[n-2]\right] \leftrightarrow \frac{0.81z^{-2}z}{z-0.9} = \frac{0.81}{z(z-0.9)}, \quad |z| > 0.9$$

(d) $$0.9^n u[-n-1] = -\left(-0.9^n u[-n-1]\right) \leftrightarrow \frac{-z}{z-0.9}, \quad |z| < 0.9$$

(e) $$0.9^{-n} u[n+1] = 0.9\left(\frac{1}{0.9}\right)^{n+1} u[n+1] \leftrightarrow 0.9\frac{z^2}{z-1/0.9} = \frac{0.9z^2}{z-1.111}, |z| < 1.111$$

(f) $$0.9^n u[-n] \leftrightarrow \sum_{n=-\infty}^{0} 0.9^n z^{-n} = \sum_{n=0}^{\infty} (0.9^{-1} z)^n = \frac{1}{1-0.9^{-1}z} = \frac{-0.9}{z-0.9}, |z| < 0.9$$

3.50 Given the general system transfer function

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{a_0 + a_1 z^{-1} + a_1 z^{-1} + \cdots + a_N z^{-N}}$$

show that the system is causal if $a_0 \neq 0$. (Consider the impulse response.)

Solution

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}}$$

$$= a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N} \overline{\big)\, b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}$$

$$H(z) = \frac{b_0}{a_0} + \frac{1}{a_1}\left[b_1 - \frac{a_1 b_0}{a_0} \right] z^{-1} + \cdots$$

$$\Rightarrow h[n] = \frac{b_0}{a_0}\delta[n] + \frac{1}{a_1}\left[b_1 - \frac{a_1 b_0}{a_0} \right]\delta[n-1] + \cdots$$

If a_0 is not zero, the impulse response has no terms that occur before $t = 0$. The system is causal. If a_0 is zero, the first term in the quotient of $H(z)$ is $\frac{b_0}{a_1} z$. This would result in a term in the impulse response of $\frac{b_0}{a_1}\delta[n+1]$ and the system would be non-causal.