1. An analog filter has a cutoff frequency of $\omega_c$. To make a digital filter using the BLT, this cutoff frequency is prewarped to a new frequency called $\omega_{\text{prewarp}}$. What happens to $\omega_c$ and $\omega_{\text{prewarp}}$ as the sample frequency $f_s$ gets very large? Do they get close together or further apart? Justify your answer.

2. The BLT was derived from trapezoidal integration. Simpson's rule provides a method of approximating an integral by determining the area under a parabola instead of a trapezoid. Thus if the figure below represents a sampled parabola, the area between $(n-2)T$ and $nT$ is given by

$$\text{Area} = \frac{2T}{3} [x(n-2)T + 4x(n-1)T + x(nT)]$$

Use this equation to derive an expression for the "parabolic transform" in $z$.

3. Write Pade's equations for the numerator and denominator coefficients of a filter which has $N = 3$ and $M = 2$ if the ideal filter is given by:

$$h(n) = \{0.25, 0.5, 1.0, 0.5, 0.25, 0.125, 0.0625, 0.03125, \ldots\}$$

It is not necessary to solve the equations.
4. Circle those filters which have the characteristics shown. Note that in some cases more than one answer should be circled to be correct.
   A) Flat pass band (low pass filter).
      Butterworth   Chebyshev   Elliptic
   B) Monotonic stop band (low pass filter).
      Butterworth   Chebyshev   Elliptic
   C) Fastest transition band for a given order.
      Butterworth   Chebyshev   Elliptic
   D) All pole analog filters.
      Butterworth   Chebyshev   Elliptic
   E) Minimizes the RMS error over all bands.
      Butterworth   Chebyshev   Elliptic

5. What are frequency transformations and what are they used for?

6. A lowpass Butterworth filter is to meet the following frequency specifications:
   Passband 0Hz to 150Hz with a ripple less than .05
   Stopband 200Hz to infinity with a ripple less than .01.
   Find the minimum order Butterworth filter to satisfy these specifications.

7. FIR filters
   A) have linear phase
   B) are lower in order than elliptic filters which meet the same specifications
   C) may have multiple zeros at the origin
   D) may have poles at the Z = -1 point
   E) are always stable
   F) All of the above
   G) None of the above

8. IIR filters
   A) have multiple feedback terms
   B) may not have poles at the origin
   C) may be unstable
   D) usually have linear phase
   E) have all positive impulse response terms
   F) All of the above
   G) None of the above

9. Lowpass Elliptic filters:
   A) are flat in the passband
   B) have poles at the origin
   C) are generally lower order than Butterworth filters that meet similar specifications.
   D) are monotonic in the stopband
   E) have feedback terms in the difference equation
   F) All of the above
10. The BLT
   A) Produces a stable digital filter from a stable analog filter.
   B) Maps and infinite frequency space in the S-plane to a finite frequency space in the z-plane.
   C) Cannot be used for analog filters that are not band limited.
   D) Can be derived from trapezoidal integration.
   E) All of the above.
   F) None of the above.

11. An IIR filter has been designed using the bilinear transform. If the final design that was
implemented has a cutoff frequency of 200 Hz and the sample frequency was 600 Hz, what was the
cutoff frequency of the analog filter **BEFORE** it was prewarped. Show your calculations.

12. In using Pade's method of direct design or Prony's method of direct design we begin by
writing a transfer function for an IIR filter along with a set of difference equations as shown
below:

\[ H(z) = \frac{\sum_{k=0}^{M} b_k z^{M-k}}{z^N + \sum_{k=1}^{N} a_k z^{N-k}} \]

\[ h(k) = b_k - a_1 h(k-1) - a_2 h(k-2) - \cdots - a_N h(k-N) \quad \text{for} \quad 0 \leq k \leq M \]
and
\[ h(k) = -a_1 h(k-1) - a_2 h(k-2) - \cdots - a_N h(k-N) \quad \text{for} \quad k > M \]

Explain how the two methods are the same and how they are different.

13. Find the pole and zero locations for a simple resonator which has a center frequency of
400Hz, a bandwidth of 20Hz, and a sample frequency of 3000Hz. Show all work. Put your
results in rectangular format.