EE 311
Fourier Series to Fourier transform

The exponential Fourier series is given by

$$f(t)=\sum_{k=-\infty}^{\infty} c_k e^{j\omega_0 kt} \quad \text{with} \quad c_k=\frac{1}{T_o}\int_{T_0} f(t)e^{-jk\omega_0 t}dt$$

The variable k is an integer and the increment in discrete frequency from one value of k to the next is $(k+1)\omega_0 - k\omega_0 = \Delta\omega = \omega_0$.

Since $\omega_0 = 2\pi/T_0$, we have

$$\Delta\omega=\lim_{T_0\to\infty}(2\pi/T_0)=d\omega$$

Also, the quantity $k\omega_0 = 2\pi k/T_0$ approaches $kd\omega$ as T_0 becomes infinite. Since k is infinitely variable over integer values, the product $kd\omega$ becomes the continuous frequency variable ω.

We can rewrite c_k as

$$c_k=\lim_{T_0\to\infty}\frac{1}{2\pi}\frac{2\pi}{T_0}\int_{-T_0/2}^{T_0/2} f(t)e^{-(jk2\pi/T_0)t}dt = \frac{1}{2\pi}\left[\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt\right]d\omega$$

The function in brackets is defined as the Fourier transform and can be written as

$$F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$ Fourier Transform

$$c_k=\frac{1}{2\pi}F(\omega)d\omega$$

Putting this value of c_k into the equation for the exponential Fourier series gives

$$f(t)=\frac{1}{2\pi}\sum_{k=-\infty}^{\infty}F(\omega)e^{jk\omega_0 t}d\omega$$

But with T_0 going to ∞ and ω_0 going to zero the term $k\omega_0$ goes to ω and the summation becomes an integral. This gives

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}d\omega$$ Inverse Fourier Transform