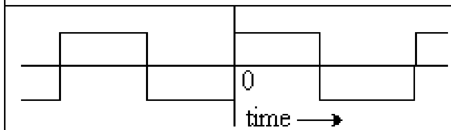
# Fourier transform summary

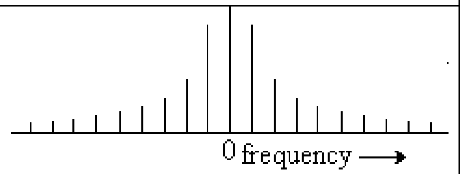
## Fourier Series

The Fourier series expresses "almost any" periodic function in terms of an infinite sum of sinusoids. The time function is continuous and the sinusoids have discrete frequency values.




$$f(t)=\sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

Continuous in time
Periodic

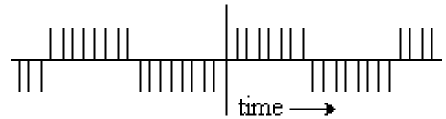



$$C_k=\frac{1}{T}\int_T f(t)e^{-jk\omega_0 t}\,dt$$

Discrete in frequency
Not periodic

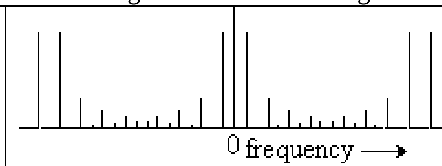
## Discrete Fourier transform

The DFT is the same as the Fourier transform if both the time and frequency variables are made discrete. For the DFT both the time and frequency domains become periodic. Since all variables are discrete, the DFT is convenient for computer calculations. The fast Fourier Transform (FFT) provides and efficient algorithm for calculating the DFT.




$$f(nT)=\frac{1}{NT}\sum_{k=0}^{N-1} F(k)e^{j\frac{2\pi kn}{N}T}$$

Discrete in time
Periodic

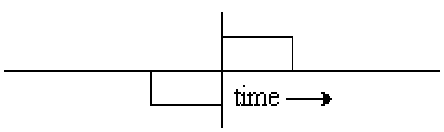



$$F(k)=T\sum_{n=0}^{N-1} f(n)e^{-j\frac{2\pi kn}{N}T}$$

Discrete in frequency
Periodic

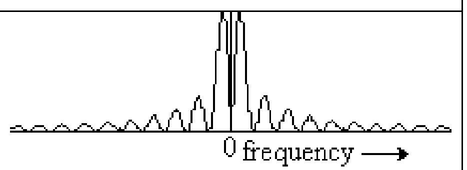
## Fourier transform

The Fourier transform can be derived from the Fourier series by allowing the period to go to infinity. The Fourier transform shows the frequency make up of non-periodic signals. The time function is continuous as is the frequency domain transform. If $C_k$ is the value of the Fourier series coefficients for a function with period T, then the Fourier transform is the limit of $TC_k$ as T goes to infinity. The Fourier transform can be "generalized" by replacing $j\omega$ with the complex variable s. The result is the Laplace transform.




$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega$$

Continuous in time
Not periodic

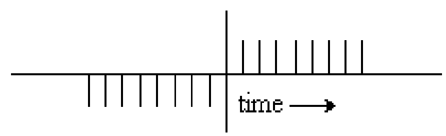



$$F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$$

Continuous in frequency
Not periodic

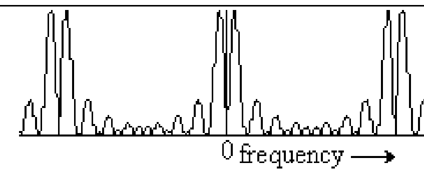
## Discrete Time Fourier transform

The DTFT is the same as the Fourier transform except that the time variable is discrete. The frequency spectrum of the resulting non-periodic discrete time signal is continuous and periodic. The DTFT can be "generalized" by allowing a new complex variable, z, to replace $e^{j\omega T}$. This results in the z-transform.




$$f(nT)=\frac{1}{2\pi}\int_T F(\omega T)e^{j\omega nT}\,d\omega$$

Discrete in time
Not periodic




$$F(\omega T)=T\sum_{n=-\infty}^{\infty} f(nT)e^{-j\omega nT}$$

Continuous in frequency
Periodic