CS 210 **November 8, 2016**
Assignment 08 **Due: December 6, 2016**
Fourier series expansion **Not accepted late**

Reminder: This is a programming project, and work on this assignment should be done individually. Assistance from other students is limited to questions about specific issues as noted in the syllabus.

In the early 1800's Joseph Fourier, who was studying the flow of heat, found that he could rewrite almost any periodic function as an infinite series of sines and cosines. His discovery made it possible to describe signals in terms of the frequency of the sines and cosine which are used to construct that signal.

If f(t) is a periodic function it can be written as an infinite sum of sines and cosines using the following equation:

$$f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cdot Cos(\omega_0 kt) + \sum_{k=1}^{\infty} b_k \cdot Sin(\omega_0 kt)$$

where

$$a_k = \frac{2}{T} \cdot \int_T f(t) \cdot Cos(\omega_0 kt) dt \quad \text{and} \quad b_k = \frac{2}{T} \cdot \int_T f(t) \cdot Sin(\omega_0 kt) dt$$

For example find the Fourier series for the square wave shown below. Period is 2π and frequency is 1/(2π).

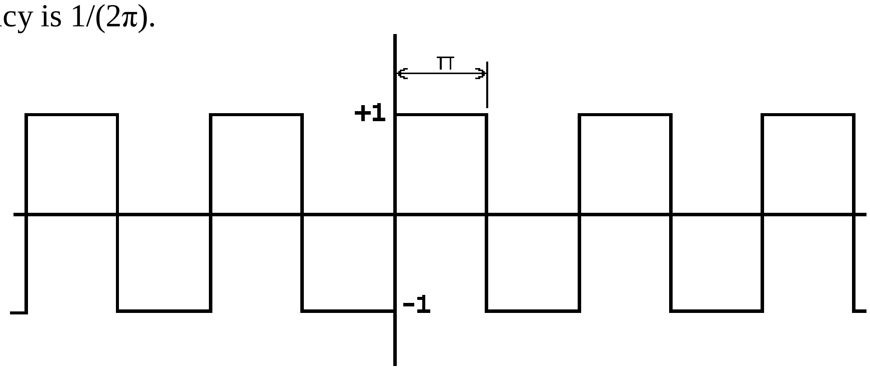

Figure 1
A square wave with a period of 2π.

To find the coefficients a_k and b_k, we need to integrate over one period. For this problem we will take the period from 0 to 2π. The equation for f(t) is:

$$f(t) = \begin{cases} +1 & 0 \le t \le \pi \\ -1 & \pi \le t \le 2\pi \end{cases}$$

The equations for a_k and b_k can be evaluated as:

$$a_k = \frac{1}{\pi}\int_0^{\pi} (+1) Cos(kt) dt + \frac{1}{\pi}\int_{\pi}^{2\pi} (-1) Cos(kt) dt \quad = 0$$

$$b_k = \frac{1}{\pi}\int_0^{\pi}(+1)Sin(kt)dt + \frac{1}{\pi}\int_{\pi}^{2\pi}(-1)Sin(kt)dt \quad = \begin{cases} 0 & k\ even \\ \dfrac{4}{k\pi} & k\ odd \end{cases}$$

Putting the values of a_k and b_k back into Fourier's equation we can write f(t) as a Fourier series.

$$f(t) = \sum_{\substack{k=1 \\ k\ odd}}^{\infty} \frac{4}{k\pi} Sin(kt) \quad \text{Square wave waveform}$$

which can be written as

$$f(t) = \frac{4}{\pi}\sin(t) + \frac{4}{3\pi}\sin(3t) + \frac{4}{5\pi}\sin(5t) + \cdots$$

Likewise, if we use a sawtooth waveform as shown in Figure 2 we get the following equation:

$$f(t) = 2\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}\sin(kt) \quad \text{Sawtooth waveform}$$

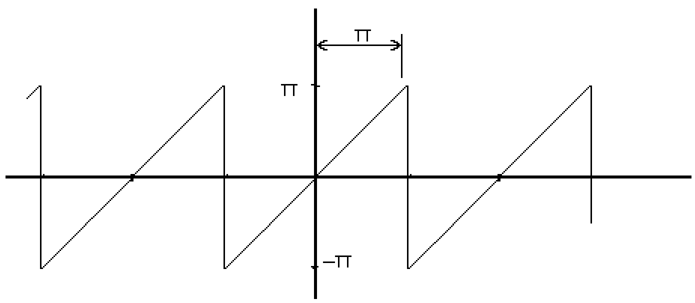

Figure 2

A sawtooth wave with a period of 2π.

Notice that for both the square wave and the sawtooth wave the sinusoids get smaller in amplitude as the number (and frequency) of sinusoids increase.

For this assignment you will write a program that will generate a file to plot the Fourier expansion of either a square wave or a sawtooth wave using either a fixed number of terms or a prescribed accuracy. The data which your program produces should be written to a file as comma separated variables (CSV) which can be read by Excel and plotted appropriately.

Your program should prompt the user for four inputs:

- Input 1: This will be the waveform type. The user is expected to enter either "square" or "sawtooth".
- Input 2: This will be a prompt for number of terms. The user will enter one number in reply. This number should be in the range 1 to 100. If it is outside of this range set it equal to 5.
- Input 3: This will be the file name where the data is to be written.
- Input 4: This will be the number of points to be used in the plot. It should be in the range 100 ≤ POINTS ≤ 1000. If the number it outside this range you should set to 200. The x-axis range will be fixed and go from -4π to +4π.

For this assignment you must dynamically allocate the data array to hold the results.

In addition to the normal zipped project file you should also include a Word document which has images of the following results:
R1: "square", 1
R2: "square", 50
R3: "square", 0.0001
R4: "sawtooth", 30
R5: "sawtooth", 0.000001

Turn in a zipped project file. Name your zipped file Asn08XXX.zip where XXX are your three initials. Upload your zipped project file to `\\cecsfp01\users\everyone\CS210.`