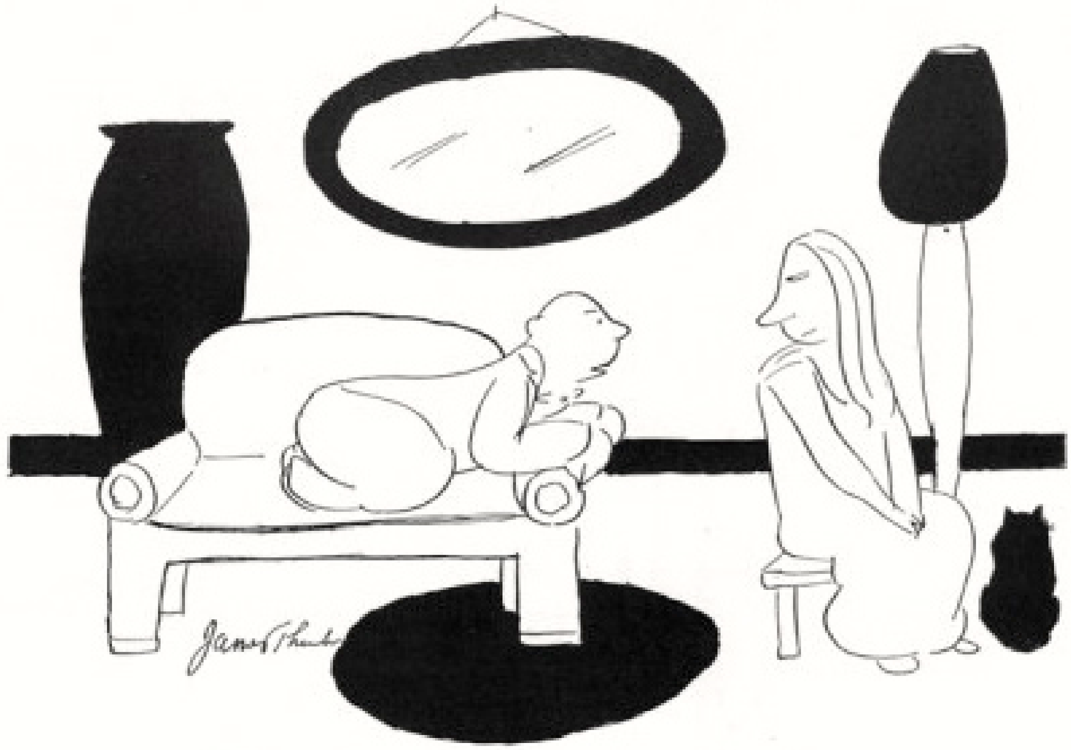


"What do you want to be inscrutable *for*, Marcia?"

EE 311 **January 16, 2019**
Notes – Second Day

Do FIR in general and Example 1.3 and plot in MATLAB

Do P 1.6 and 1.9

Example 1.3
A RLC circuit has an impulse response given by

$$h(t) = Ke^{-\alpha t}[\omega_d \sin(\omega_d t) - \alpha(1 - \cos\omega_d t)]$$

where $\alpha = 50$, $\omega_d = 1200$, and K = 0.001
Sample this impulse response at 1000 Hz and create an FIR filter from the first 30 samples. Find the frequency response of the resulting filter.

Solution
We begin by finding the first 30 terms with a sample rate of 1000 Hz.

```
fs = 1000;T = 1/fs;
f = 0:fs/2;
Len = 30;
t = 0:T:(Len-1)*T;
a = 50;wd = 1200;K = 0.001;
h = K*exp(-a*t).*(wd*sin(wd*t) ...
               - a*(1 - cos(wd*t)));
figure(1);clf;
stem(t, h);
axis([0 Len*T -2 5.2]);
xlabel('Time in seconds');
ylabel('h');
title('Sampled impulse response');
```

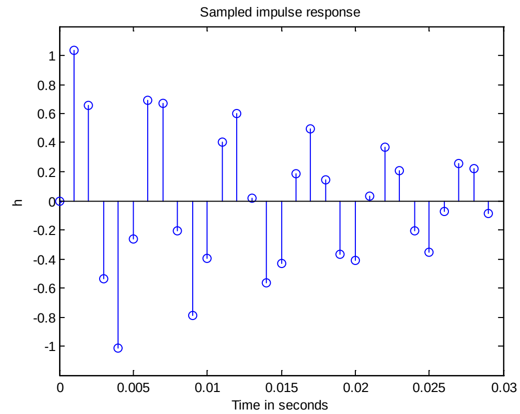


Figure 1.12

Calculating and plotting the first 30 samples of $h(t)=Ke^{-\alpha t}[\omega_d \sin(\omega_d t)-\alpha(1-\cos\omega_d t)]$

The difference equation will have the same form as (1.17).

$$v_o(nT) = h(0)v_i(nT) + h(1)v_i([n-1]T) + h(2)v_i([n-2]T) + \cdots + h(28)v_i([n-28]T) + h(29)v_i([n-29]T) \quad \textbf{(1.18)}$$

To find the frequency response we let $v_i(nT) = e^{j\omega nT}$ and $v_o(nT) = Ae^{j\omega nT}$ where A is a complex number. Substituting these values into (1.18) gives

$$Ae^{j\omega nT} = h(0)e^{j\omega nT} + h(1)e^{j\omega(n-1)T} + h(2)e^{j\omega(n-2)T} + \cdots + h(28)e^{j\omega(n-28)T} + h(29)e^{j\omega(n-29)T}$$

We can divide out $e^{j\omega nT}$ from both sides to get

$$A = h(0) + h(1)e^{-j\omega T} + h(2)e^{-j\omega 2T} + \cdots + h(28)e^{-j\omega 28T} + h(29)e^{-j\omega 29T}$$

This equation can be used to plot the magnitude of A versus frequency. Since the magnitude of the input is one, the magnitude of *A* is the gain for the circuit. Figure 1.13 shows the results.

```
w = 2*pi*f;
jwT = j*w*T;
mag = zeros(1, length(f));
for i = 1:Len
    mag = mag + h(i)*(exp(jwT*(1-i)));
end
mag = abs(mag);
figure(2);clf;
plot(f, mag);
```

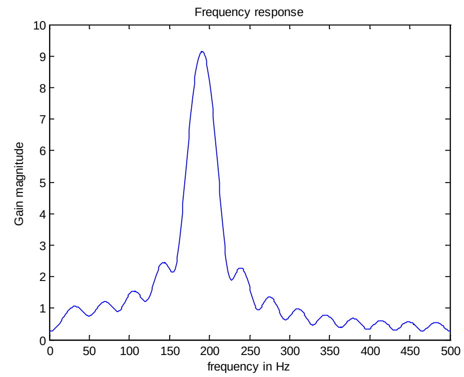


Figure 1.13
Calculating and plotting the frequency response. The frequency response shows this circuit to be a resonant circuit with a resonant frequency near 200 Hz.

The IIR and FIR filters produced in the previous examples are far from ideal and are given to illustrate the process and add some intuition as to how DSP systems work in general. Several complex design issues have been simplified in this introduction. These will be addressed in a more rigorous fashion in later chapters.

1.6 Show that the difference equation for the second order IIR filter Figure P1.6 is given by

$$y_k = b_0 x_k + b_1 x_{k-1} + b_2 x_{k-2} - a_1 y_{k-1} - a_2 y_{k-2}$$

where y_k is the output variable at time k and u_k is the input variable at time k.

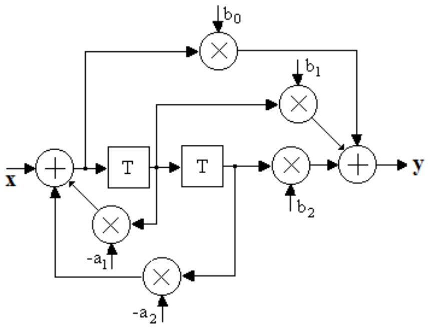
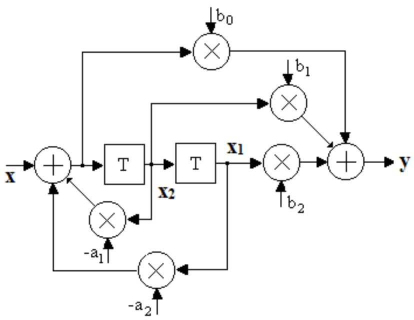


Figure P1.6
An IIR filter.

Solution



$$y(k) = b_2 x_1(k) + b_1 x_2(k) + b_0 x_2(k+1)$$

$$x_1(k+1) = x_2(k)$$

$$x_2(k+1) = -a_2 x_1(k) - a_1 x_2(k)$$

k	x(k)	x2(k+1)	x2(k)	x1(k)	y(k)
0	1	1	0	0	b_0
1	0	$-a_1$	1	0	$-b_0a_1+b_1$
2	0	$-a_2+a_1^2$	$-a_1$	1	$b_2-b_1a_1+b_0(-a_2+a_1^2)$
3	0	$a_1a_2 + a_1a_2-a_1^3$	$-a_2+a_1^2$	$-a_1$	$-b_2a_1+b_1(-a_2+a_1^2)+b_0(a_1a_2 + a_1a_2-a_1^3)$

We can also get the impulse response from the difference equation:

$$y_k = b_0 x_k + b_1 x_{k-1} + b_2 x_{k-2} - a_1 y_{k-1} - a_2 y_{k-2}$$

k	x(k)	x(k-1)	x(k-2)	y(k)	y(k-1)	y(k-2)
0	1	0	0	b_0	0	0
1	0	1	0	$-b_0a_1+ b_1$	b_0	0
2	0	0	1	$b_2-b_1a_1+b_0(-a_2+a_1^2)$	$-b_0a_1+ b_1$	b_0
3	0	0	0	$-b_2a_1+b_1(-a_2+a_1^2)+b_0(a_1a_2 + a_1a_2-a_1^3)$	$b_2-b_1a_1+b_0(-a_2+a_1^2)$	$-b_0a_1+ b_1$

Since the difference equation has the same response to an impulse as does the evaluation of the state diagram the two forms must represent the same system.

1.9 Consider the difference equation given by

$$y(nT) = x(nT) + Ky(n-1)T$$

A) Find the expression for the frequency response for y/x in terms of K. Take T = 1.
B) Use MATLAB® to plot the frequency response for two cases: K = 0.8 and K = 1/0.8 = 1.25. What is the same and what is different about these two responses?
C) Use MATLAB® to plot the impulse response for two cases: K = 0.8 and K = 1/0.8 = 1.25. Plot at least 10 terms.
D) What can you conclude from the impulse response of these two difference equations that is not evident from the frequency response?

Solution

A) The difference equation is:

$$y(nT) = x(nT) + Ky(n-1)T$$

Let $x(nT) = e^{j\omega nT}$, $y(nT) = Ae^{j\omega nT}$ and $y(n-1)T = Ae^{j\omega(n-1)T}$

This gives

$$Ae^{j\omega nT} = e^{j\omega nT} + KAe^{j\omega(n-1)T}$$

Divide both sides by $e^{j\omega nT}$

$$A = 1 + KAe^{-j\omega T}$$

Solve this for *A* to get

$$A = \frac{1}{1 - Ke^{-j\omega T}}$$

$$\frac{y(e^{j\omega nT})}{x(e^{j\omega nT})} = \frac{Ae^{j\omega nT}}{e^{j\omega nT}} = A$$

Applying Euler's identity we get

$$\frac{y(e^{j\omega nT})}{x(e^{j\omega nT})} = \frac{1}{1 - K\cos(\omega T) + jK\sin(\omega T)}$$

The magnitude response is:

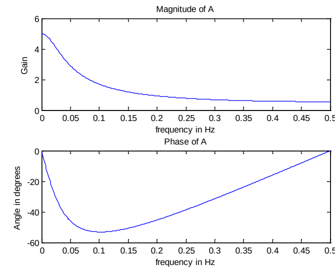
$$\left|\frac{ye^{j\omega nT}}{xe^{j\omega nT}}\right| = \frac{1}{\sqrt{(1 - K\cos(\omega T)^2 + K^2\sin^2(\omega T)}}$$

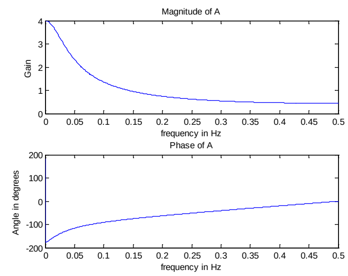
The phase response is:

$$\theta(\omega T) = 0 - \tan^{-1}\left(\frac{K\sin(\omega T)}{1 - K\cos(\omega T)}\right)$$

B) The magnitude plots have the same shape but when K = 0.8 the magnitude is overall slightly higher. If both plots were normalized to a gain of unity at 0 Hz they would be the same. The phase plots are very different. For K = 0.8 the phase curve begins and ends at 0°. It reaches a maximum of about -50° at about 0.1 Hz. When K = 1.25 the phase curve begins at -180° and ends at 0°. The total phase changes is about 180°.

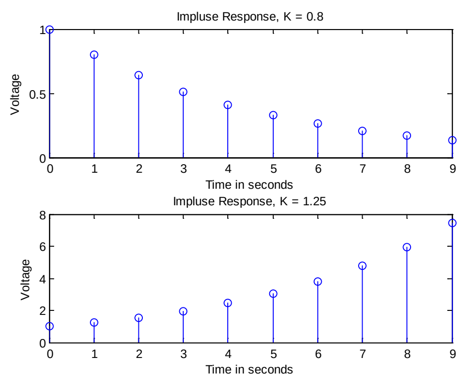
```
fs = 1;T = 1/fs;
f = 0:.001:fs/2;
w = 2*pi*f;
K = 0.8;
A = 1./(1-K*exp(-j*w*T));
figure(1);clf;
subplot(2, 1, 1);
plot(f, abs(A));
title('Magnitude of A, K = 0.8');
xlabel('frequency in Hz');
ylabel('Gain');
subplot(2, 1, 2);
plot(f, angle(A)*180/pi);
title('Phase of A, K = 0.8');
xlabel('frequency in Hz');
ylabel('Angle in degrees');
```





C)

```
x = [1 zeros(1, 9)]; %impulse in
y = zeros(1, 10);
y(1) = x(1);
K = 0.8;
for i = 2:length(y)
    y(i) = x(i) + K*y(i-1);
end
figure(4);clf;
fs = 1;T = 1/fs;
nT = (0:9)*T;
subplot(2, 1, 1);
stem(nT, y);
title('Impulse Response, K = 0.8');
xlabel('Time in seconds');
ylabel('Voltage');
```



D) When K = 1.25 the system is unstable.

```
%Example1_3
fs = 1000;T = 1/fs;
f = 0:fs/2;
Len = 30;
t = 0:T:(Len-1)*T;
a = 50;wd = 1200;K = 0.001;
h = K*exp(-a*t).*(wd*sin(wd*t) ...
              - a*(1 - cos(wd*t)));
figure(1);clf;
stem(t, h);
axis([0 Len*T -2 5.2]);
xlabel('Time in seconds');
ylabel('h');
title('Sampled impulse response');
w = 2*pi*f;
jwT = j*w*T;
mag = zeros(1, length(f));
for i = 1:Len
    mag = mag + h(i)*(exp(jwT*(1-i)));
end
mag = abs(mag);
figure(2);clf;
plot(f, mag);
xlabel('frequency in Hz');
ylabel('magnitude');
title('Frequency response');
```

```
%Problem1_9
fs = 1;T = 1/fs;
f = 0:.001:fs/2;
w = 2*pi*f;
K = 0.8; %1.25
A = 1./(1-K*exp(-j*w*T));
figure(1);clf;
subplot(2, 1, 1);
plot(f, abs(A));
title(['Magnitude of A, K = ' num2str(K)]);
xlabel('frequency in Hz');
ylabel('Gain');
subplot(2, 1, 2);
plot(f, angle(A)*180/pi);
title(['Phase of A, K = ' num2str(K)]);
xlabel('frequency in Hz');
ylabel('Angle in degrees');
%
x = [1 zeros(1, 9)]; %impulse in
y = zeros(1, 10);
y(1) = x(1);
for i = 2:length(y)
    y(i) = x(i) + K*y(i-1);
end
figure(2);clf;
fs = 1;T = 1/fs;
nT = (0:9)*T;
subplot(2, 1, 1);
stem(nT, y);
title(['Impulse Response, K = ' num2str(K)]);
xlabel('Time in seconds');
ylabel('Voltage');

x = [1 zeros(1, 9)]; %impulse in
y = zeros(1, 10);
y(1) = x(1);
for i = 2:length(y)
    y(i) = x(i) + K*y(i-1);
end
figure(2);clf;
fs = 1;T = 1/fs;
nT = (0:9)*T;
subplot(2, 1, 1);
stem(nT, y);
title(['Impulse Response, K = ' num2str(K)]);
xlabel('Time in seconds');
ylabel('Voltage');
```