"If you can keep a secret, I'll tell you how my husband died."

EE 311 **January 18, 2019**

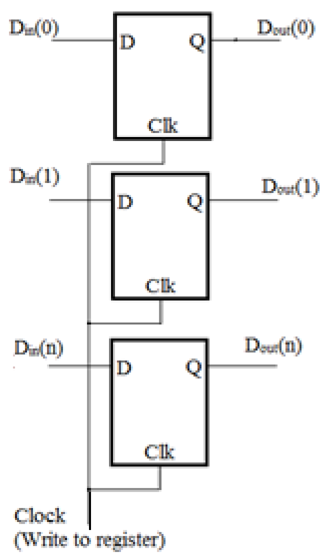
Notes – Third Day

Chapter 2

- Impulse in discrete time is one unit high – does not go off to infinity.
- analogous operations

Continuous Time	**Discrete Time**
1. $\int_{-\infty}^{t} x(\tau)d\tau$	$\sum_{k=-\infty}^{n} x[k]$ (2.15)
2. $\frac{dx(t)}{dt}$	$\frac{x[n]-x[n-1]}{T}$ (2.16), or $x[n]-x[n-1]$ (2.17)
3. $x(t)\delta(t) = x(0)\delta(t)$	$x[n]\delta[n] = x[0]\delta[n]$ (2.18)
4. $\delta(t) = \frac{du(t)}{dt}$	$\delta[n] = u[n] - u[n-1]$ (2.19)
5. $u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$	$u[n] = \sum_{k=-\infty}^{n} \delta[k]$ (2.20)

- What is a time delay?



- Convolution – do example as superposition

  In Class exercise

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n]*h[n].$$

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Convolution

Suppose a discrete system has an impulse response given by
h = {1 2 4 2}

A) If the input is given by x = {1, 4, 6} fill in the convolution table below to show the output y.

B) Write a MATLAB m-file to produce the same result.

EE 311 **January 18, 2019**
Convolution SOLUTION

Suppose a discrete system has an impulse response given by
h = {1 2 4 2}

A) If the input is given by x = {1, 4, 6} fill in the convolution table below to show the output y.

B) Write a MATLAB m-file to produce the same result.

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
x1	1	2	4	2	0	0	0	0	0
x2	0	4	8	16	8	0	0	0	0
x3	0	0	6	12	24	12	0	0	0
Sum	1	6	18	30	32	12	0	0	0

```
%ConvolutionExmp.m
x = [1 4 6];
h = [1 2 4 2];
y = conv(h, x);
disp(y);
%Displays 1     6     18     30     32     12
```

• Do problem 2.25

2.25. Write a difference equation model for the system shown in Figure P2.24.

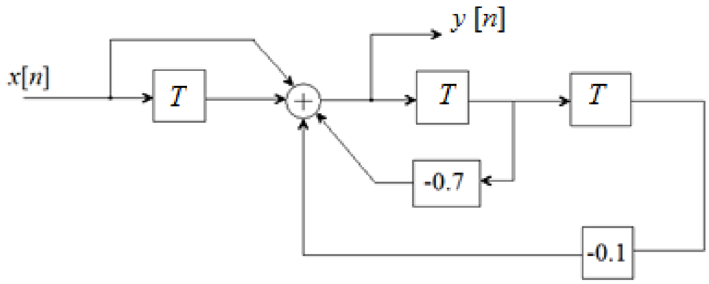


Figure P2.25

Solution P2.25

Recognize that the output of each block is the input to that block delayed by one sample period. Write the equation for the output of the summing junction.

$y[n] = -0.7y[n-1] - 0.1y[n-2] + x[n] + x[n-1]$

• Do 1.6

1.6 Show that the difference equation for the second order IIR filter Figure P1.6 is given by

$$y_k = b_0 x_k + b_1 x_{k-1} + b_2 x_{k-2} - a_1 y_{k-1} - a_2 y_{k-2}$$

where y_k is the output variable at time k and u_k is the input variable at time k.

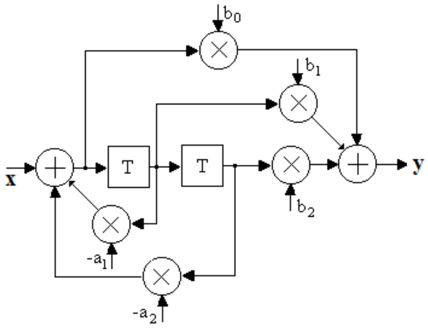
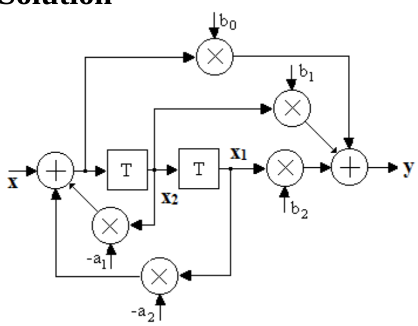


Figure P1.6
An IIR filter.

Solution



$$y(k) = b_2 x_1(k) + b_1 x_2(k) + b_0 x_2(k+1)$$
$$x_1(k+1) = x_2(k)$$
$$x_2(k+1) = -a_2 x_1(k) - a_1 x_2(k)$$

k	x(k)	$x_2(k+1)$	$x_2(k)$	$x_1(k)$	y(k)
0	1	1	0	0	b_0
1	0	$-a_1$	1	0	$-b_0a_1+b_1$
2	0	$-a_2+a_1^2$	$-a_1$	1	$b_2-b_1a_1+b_0(-a_2+a_1^2)$
3	0	$a_1a_2 + a_1a_2-a_1^3$	$-a_2+a_1^2$	$-a_1$	$-b_2a_1+b_1(-a_2+a_1^2)+b_0(a_1a_2 + a_1a_2-a_1^3)$

We can also get the impulse response from the difference equation:

$$y_k = b_0 x_k + b_1 x_{k-1} + b_2 x_{k-2} - a_1 y_{k-1} - a_2 y_{k-2}$$

k	x(k)	x(k-1)	x(k-2)	y(k)	y(k-1)	y(k-2)
0	1	0	0	b_0	0	0
1	0	1	0	$-b_0a_1+ b_1$	b_0	0
2	0	0	1	$b_2-b_1a_1+b_0(-a_2+a_1^2)$	$-b_0a_1+ b_1$	b_0
3	0	0	0	$-b_2a_1+b_1(-a_2+a_1^2)+b_0(a_1a_2 + a_1a_2-a_1^3)$	$b_2-b_1a_1+b_0(-a_2+a_1^2)$	$-b_0a_1+ b_1$

Since the difference equation has the same response to an impulse as does the evaluation of the state diagram the two forms must represent the same system.

Chapter 3

• Fourier series

Fourier Series Form	Equation
Exponential	$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t};$ $c_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt = \|c_k\|e^{j\theta_k},\ c_{-k} = c_k^*$
Combined Trigonometric	$x(t) = c_0 + \sum_{k=1}^{\infty} 2\|c_k\|\cos(k\omega_0 t + \theta_k)$
Trigonometric	$x(t) = a_0 + \sum_{k=1}^{\infty}[a_k\cos(k\omega_0 t) + jb_k\sin(k\omega_0 t)],$ $a_0 = c_0 = \frac{1}{T_0}\int_{T_0} x(t)dt, \quad 2c_k = a_k - jb_k$ $a_k = \frac{2}{T_0}\int_{T_0} x(t)\cos(k\omega_0 t)dt$ and $b_k = \frac{2}{T_0}\int_{T_0} x(t)\sin(k\omega_0 t)dt$

The Fourier series uses the trigonometric functions as an orthogonal basis set. The trigonometric form of the Fourier series is given by the equation:

$$f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cdot Cos(\omega_0 kt) + \sum_{k=1}^{\infty} b_k \cdot Sin(\omega_0 kt) \qquad \textbf{2.4}$$

where

$$a_k = \frac{2}{T}\cdot\int_T f(t)\cdot Cos(\omega_0 kt)dt \quad \text{and} \quad b_k = \frac{2}{T}\cdot\int_T f(t)\cdot Sin(\omega_0 kt)dt$$

Example 2.1 Find the Fourier series for the square wave shown below.

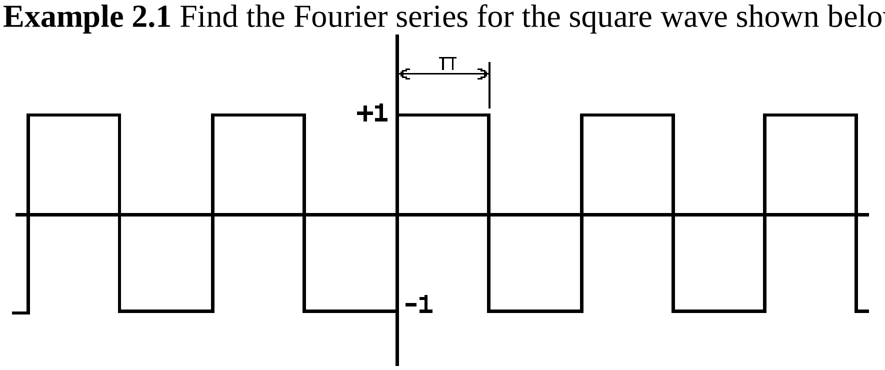


Figure E2.1-1

A square wave with a period of 2π.

Solution:

To find the coefficients a_k and b_k, we need to integrate over one period. For this problem we will take the period from 0 to 2π. The equation for f(t) is:

$$f(t) = \begin{cases} +1 & 0 \le t \le \pi \\ -1 & \pi \le t \le 2\pi \end{cases}$$

The equations for a_k and b_k can be evaluated as:

$$a_k = \frac{1}{\pi}\int_0^{\pi}(+1)Cos(kt)dt + \frac{1}{\pi}\int_{\pi}^{2\pi}(-1)Cos(kt)dt \quad = 0$$

$$b_k = \frac{1}{\pi}\int_0^{\pi}(+1)Sin(kt)dt + \frac{1}{\pi}\int_{\pi}^{2\pi}(-1)Sin(kt)dt \quad = \begin{cases} 0 & k\ even \\ \dfrac{4}{k\pi} & k\ odd \end{cases}$$

We can then write f(t) as a Fourier series.

$$f(t) = \sum_{\substack{k=1 \\ k\ odd}}^{\infty} \frac{4}{k\pi} Sin(kt)$$

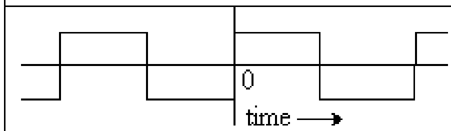
```
%Fourier.m
% This program plots terms of the Fourier series for a square wave.
terms = 5;
sum = zeros(1,201);
t = linspace(-pi,pi,201);
for k=1:2:2*terms
   y = (4/(k*pi))*sin(k*t);
   sum = sum + y;
end
figure(1);clf;
plot(t, sum)
axis([-pi pi -1.2 1.2])
```

```matlab
%Fourier.m
% This program plots terms of the Fourier series for a square wave.
x1 = [-2*pi -pi -pi 0 0 pi pi 2*pi];
y1 = [1 1 -1 -1 1 1 -1 -1];
terms = [1 3 5 10 50 100];           %Vector of number of terms in each plot
for indx = 1:6                       %6 plots
  sum = zeros(1,1024);               %1024 terms
  t = linspace(-2*pi,2*pi,1024);     %t goes from -2Pi to +2Pi
  for k=1:2:2*terms(indx)
      y = 4/(k*pi)*sin(k*t);    %Calculate next term
      sum = sum + y;         %Add to series sum
  end
  figure(indx);clf;
  plot(t, sum, 'LineWidth', 1);
  hold on;
  plot(x1, y1, 'LineWidth', 1, 'Color','r');
  xlabel('Time');
  ylabel('Amplitude');
  sTitle = strcat(['Fourier Series with ' int2str(terms(indx)) ' terms']);
  title(sTitle);
  axis([-2*pi 2*pi -1.4 1.4])
end
```

Fourier transform summary

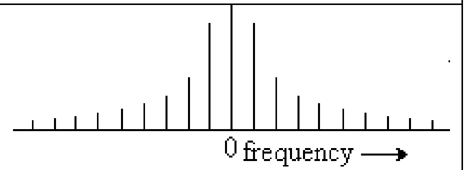
Fourier Series

The Fourier series expresses "almost any" periodic function in terms of an infinite sum of sinusoids. The time function is continuous and the sinusoids have discrete frequency values.



$$f(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

Continuous in time
Periodic

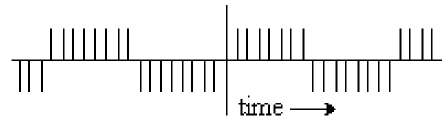


$$C_k = \frac{1}{T}\int_T f(t) e^{-jk\omega_0 t}\, dt$$

Discrete in frequency
Not periodic

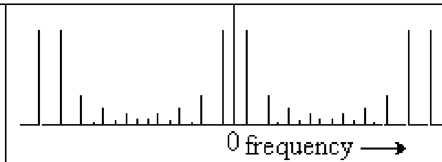
Discrete Fourier transform

The DFT is the same as the Fourier transform if both the time and frequency variables are made discrete. For the DFT both the time and frequency domains become periodic. Since all variables are discrete, the DFT is convenient for computer calculations. The fast Fourier Transform (FFT) provides and efficient algorithm for calculating the DFT.



$$f(nT) = \frac{1}{NT}\sum_{k=0}^{N-1} F(k) e^{j\frac{2\pi kn}{N}T}$$

Discrete in time
Periodic

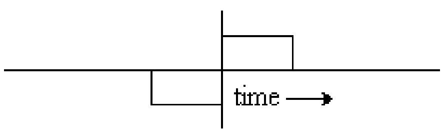


$$F(k) = T\sum_{n=0}^{N-1} f(n) e^{-j\frac{2\pi kn}{N}T}$$

Discrete in frequency
Periodic

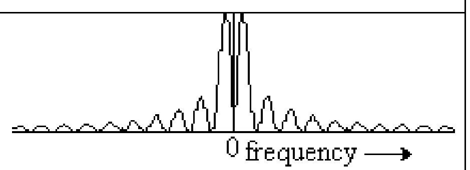
Fourier transform

The Fourier transform can be derived from the Fourier series by allowing the period to go to infinity. The Fourier transform shows the frequency make up of non-periodic signals. The time function is continuous as is the frequency domain transform. If C_k is the value of the Fourier series coefficients for a function with period T, then the Fourier transform is the limit of TC_k as T goes to infinity. The Fourier transform can be "generalized" by replacing $j\omega$ with the complex variable s. The result is the Laplace transform.



$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t}\, d\omega$$

Continuous in time
Not periodic

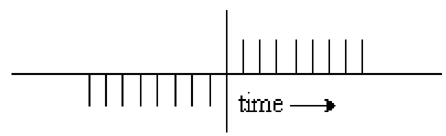


$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt$$

Continuous in frequency
Not periodic

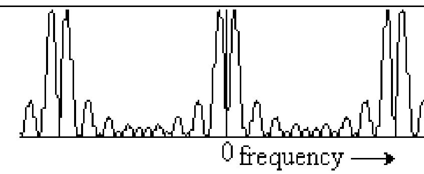
Discrete Time Fourier transform

The DTFT is the same as the Fourier transform except that the time variable is discrete. The frequency spectrum of the resulting non-periodic discrete time signal is continuous and periodic. The DTFT can be "generalized" by allowing a new complex variable, z, to replace $e^{j\omega T}$. This results in the z-transform.



$$f(nT) = \frac{1}{2\pi}\int_T F(\omega T) e^{j\omega nT}\, d\omega$$

Discrete in time
Not periodic



$$F(\omega T) = T\sum_{n=-\infty}^{\infty} f(nT) e^{-j\omega nT}$$

Continuous in frequency
Periodic