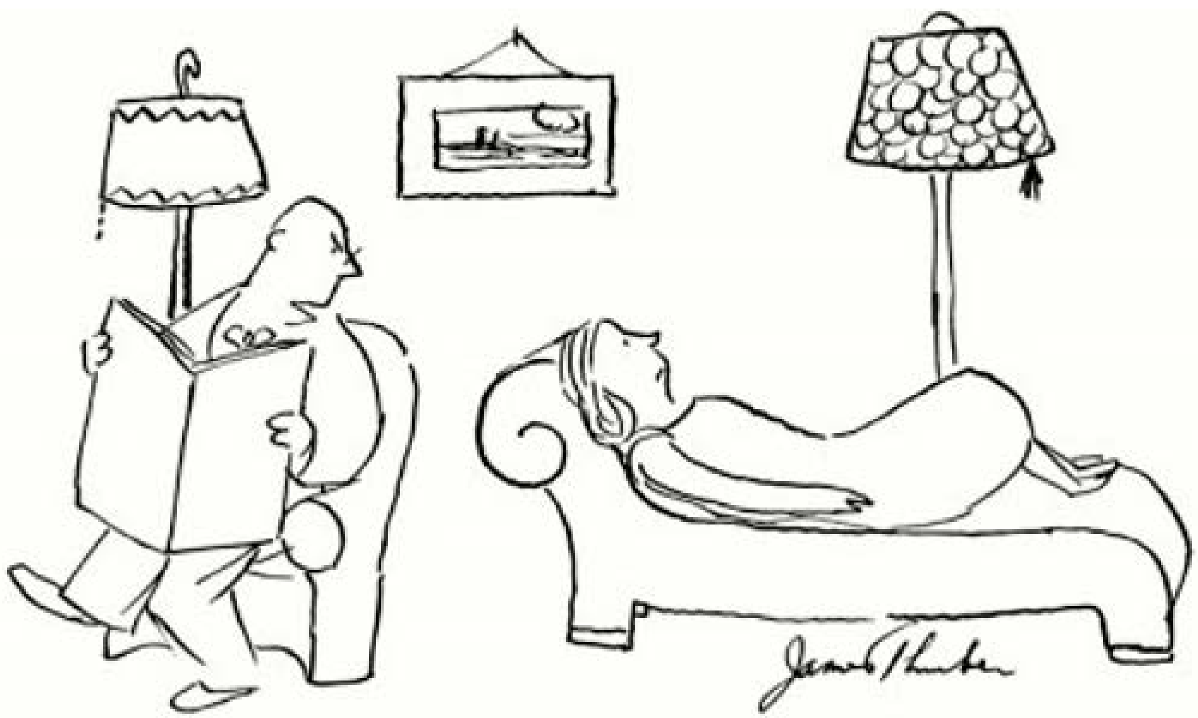


"I do love you. I just don't feel like talking military tactics with you."

EE 311 **February 10, 2019**
Notes – 9th Day

Figures 4.1 and 4.2
Do Impulse sampling Figure 4.3

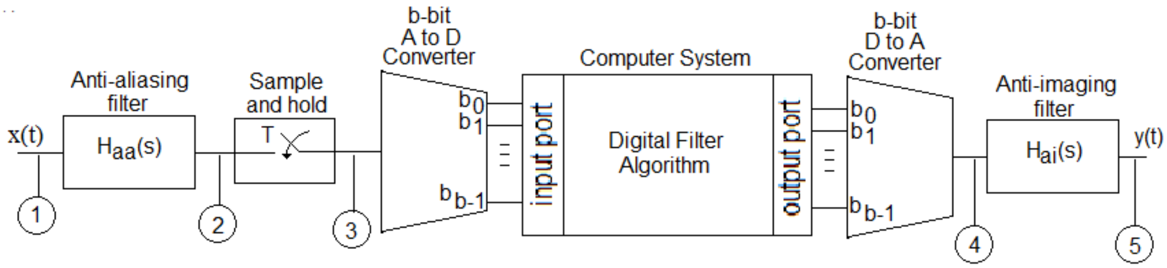


Figure 4.1
A digital filter with an interface to the continuous time domain on the input and the output.

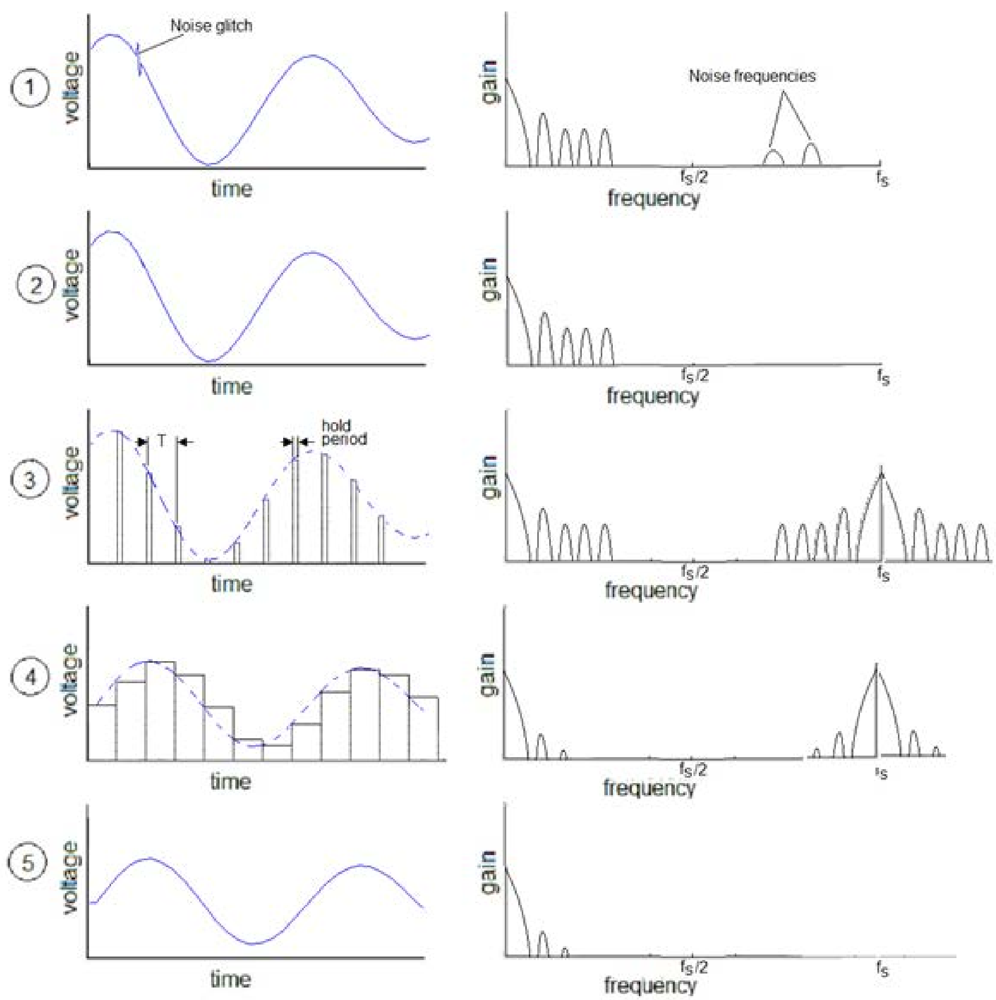


Figure 4.2
Typical time and frequency domain signals passing through a digital filter system.

$$x_s(t) = \sum_{n=0}^{\infty} x(t)\delta(t - nT)$$

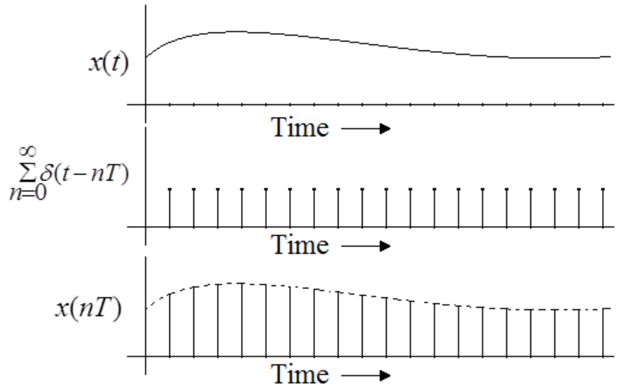


Figure 4.3
Representing a sampled signal as the product of a continuous time signal and an impulse train.

Sampling Theorem

$$x_s(t) = \sum_{n=0}^{\infty} x(t)\delta(t - nT)$$

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \overset{F}{\longleftrightarrow} \omega_S \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_S),$$

where $\omega_s = 2\pi / T$ is the sampling frequency in radians/second.

$$X_s(\omega) = \frac{1}{2\pi} X(\omega) * \left[\omega_s \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_S) \right] = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(\omega) * \delta\left(\omega - k\omega_S\right).$$

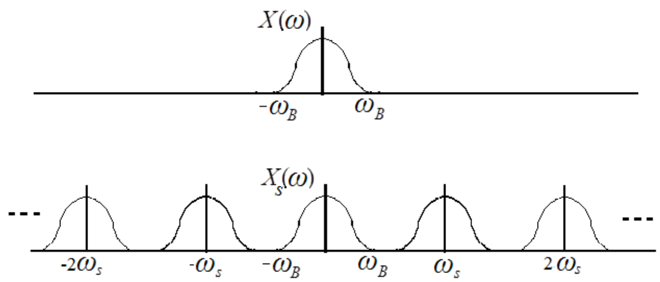
But

$$X(\omega) * \delta(\omega - k\omega_S) = X(\omega - k\omega_S). \qquad (4.3)$$

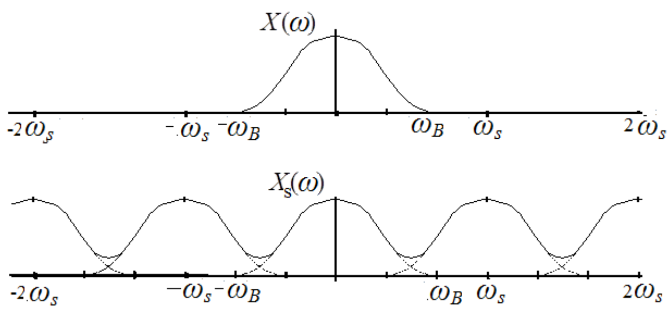
Equation (4.3) gives us the frequency domain characteristics of a sampled signal in terms of the unsampled signal and the sampling frequency.

Shannon's sampling theorem

A function of time $x(t)$, which contains no frequency components greater than f_M hertz is determined uniquely by the set of values of $x(t)$ taken at any set of points spaced $T_M/2$ seconds apart ($T_M = 1/f_M$).

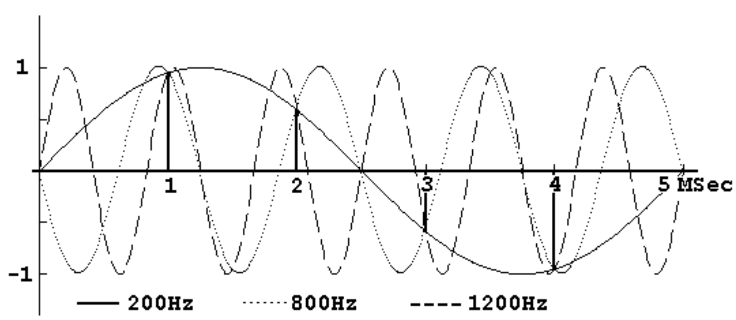


(a) $\omega_B < \omega_s/2$.



(b) $\omega_B > \omega_s/2$

Figure 4.4
(a) a continuous time signal's frequency response. The signal is limited to band $|\omega| < \omega_B$ and $\omega_B < \omega_s/2$.
(b) This is the same signal as in (a) except $\omega_B > \omega_s/2$, so that the spectral images overlap. Information is lost.



We can more formally define aliasing in the following manner:

For any sinusoid of frequency *f* that is sampled at f_s, there are an infinite number of other sinusoids of frequency $|f + kf_s|$ which will provide the same set of sample values. In this relationship, k is any non-zero integer.

Example 4.1
A sinusoid at 8 KHz is being sampled at 44.1 KHz. Find the value of the lowest five sinusoidal frequencies which can alias as 8 KHz.

Solution
Imagine an idealized frequency spectrum that runs from a gain of 1 at 0 Hz to a gain of 0 at $f_s/2$ Hz. Because of sampling this spectrum will be replicated as shown in Figure 4.6. Choose some frequency f_1 within the original spectrum and draw a horizontal straight line at the gain value for this frequency. All other frequencies which have the same gain value will be aliases of this frequency. In Figure 4.5, f_1 has aliases at f_2, f_3, …f_7 …

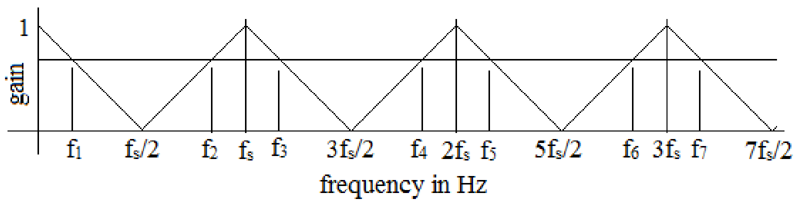


Figure 4.6
An idealized frequency spectrum used to determine aliasing frequencies.

In general, we see that an arbitrary frequency f_1 will have aliases at $kf_s \pm f_1$ where k = 1, 2, 3, … The first five aliases for 8 KHz when it is sampled at 44.1 KHz are 36.1 KHz, 52.1 KHz, 80.1 KHz, 96.1 KHz, and 120.1 KHz.

Butterworth anti-aliasing filters

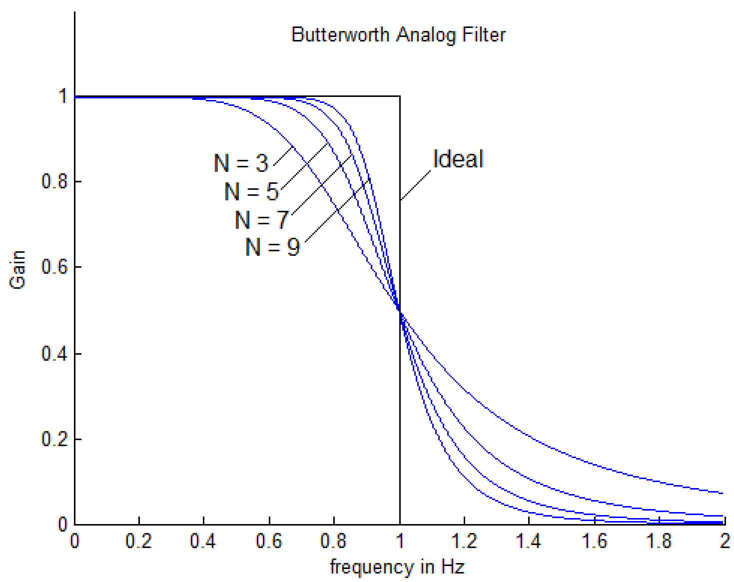


Figure 4.7
Analog Butterworth filters of order 3, 5, 7, and 9 compared to an ideal low pass filter.

Switched-capacitor filters

In Figure 4.8, if switch S_a is closed and switch S_b is open the amount of charge transferred into the capacitor is $q_a = C_1 V_a$.

If switch S_a is then opened and switch S_b is closed the amount of charge in the capacitor will be $q_b = C_1 V_b$. The voltage between points a and b is $V_a - V_b$.

The amount of charge transferred from a to b is $q_a - q_b = C_1(V_a - V_b) = \Delta q$ and this charge was transferred in an amount of time Δt.

But $\Delta q/\Delta t$ is a current. We can write

$i = \frac{\Delta q}{\Delta t} = \frac{C_1(V_a - V_b)}{\Delta t}$. But the voltage divided by the current is a resistance. This gives us

$$R_{eq} = \frac{\Delta t}{C_1}$$

If the circuit is switched periodically every Δt seconds and $f = 1/\Delta t$ we get $R_{eq} = \frac{1}{C_1 f}$

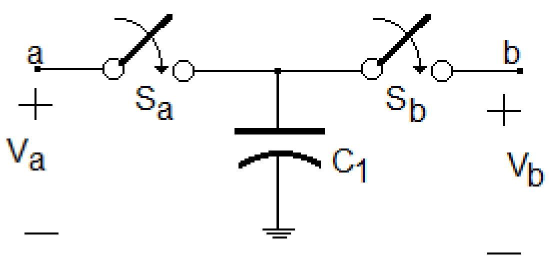


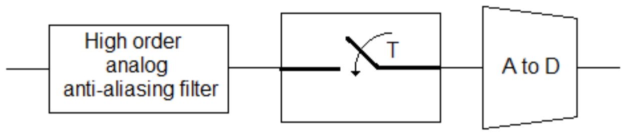
Figure 4.8
A switched capacitor can act like a resistor.

Filters constructed using this principle are called switched-capacitor filters and are commonly used as anti-aliasing filters because they can be made at low cost and high precision.

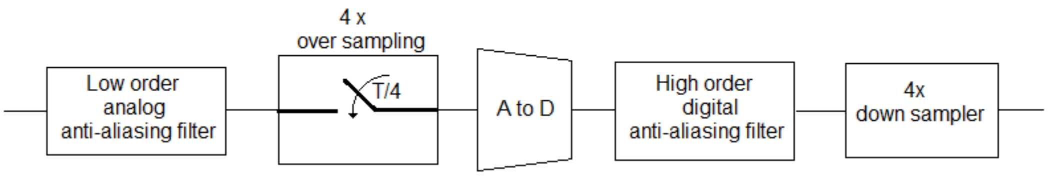
Switched capacitor filters are commonly available with orders of 2 to 8 and they can be cascaded if necessary. As a practical example the MAXIM® MAX7420 provides a 5th order low pass Butterworth response with a pass band corner frequency that is user tunable from 1 KHz to 45 KHz [2]. The device is available in a simple 8-pin package that requires two external capacitors and an external oscillator which is used to determine the corner frequency. Similar devices provide a 5th order Bessel or elliptic low pass response. All can be cascaded.

Look up data sheet at http://datasheets.maxim-ic.com/en/ds/MAX7418-MAX7425.pdf

Oversampling



A) Conventional anti-aliasing filter



B) Oversampling anti-aliasing filter

Example 4.2

A digital signal processor (DSP system with a sampling frequency f_s = 11.025 KHz requires an anti-aliasing filter which meets the following specifications:

Passband: 0 to 3.85 KHz
Passband gain: 0.95 minimum to 1.0 maximum
Stopband: 6.615 KHz to ∞
Stopband gain: 0.05 maximum

These specifications can be met using an 8th order analog Butterworth filter. Alternatively, we can meet these specifications with a 3rd order analog Butterworth filter followed by an 8th order digital Butterworth filter that is four times oversampled. Following the diagram in Figure 4.9(b) the low order analog anti-aliasing filter is a 3rd order Butterworth filter as shown
Figure 4.10.

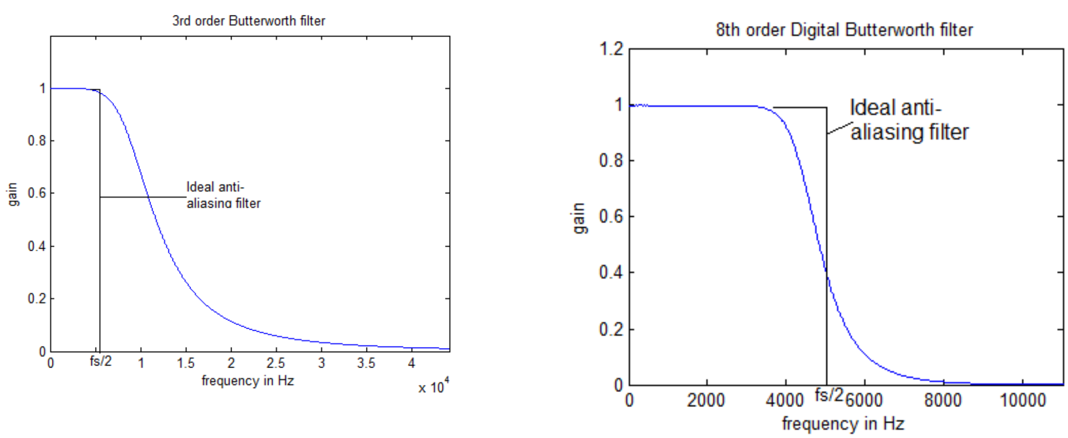


Figure 4.10
Left is a 3rd order analog anti-aliasing filter. This is followed by a 4x oversampler and an 8th order digital Butterworth filter as shown at right.

Figure 4.11 shows the results of the combined filter. The pass band and stop band are shown in exploded views to verify that specifications are met.

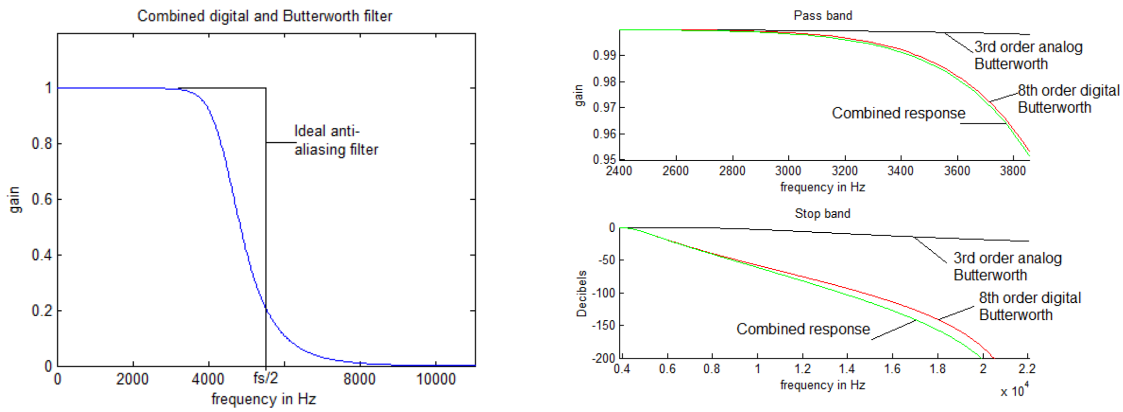
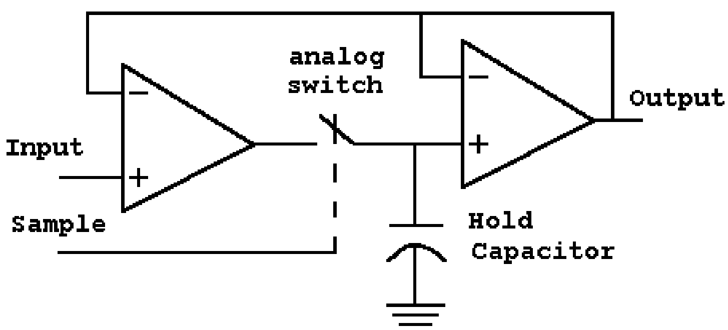
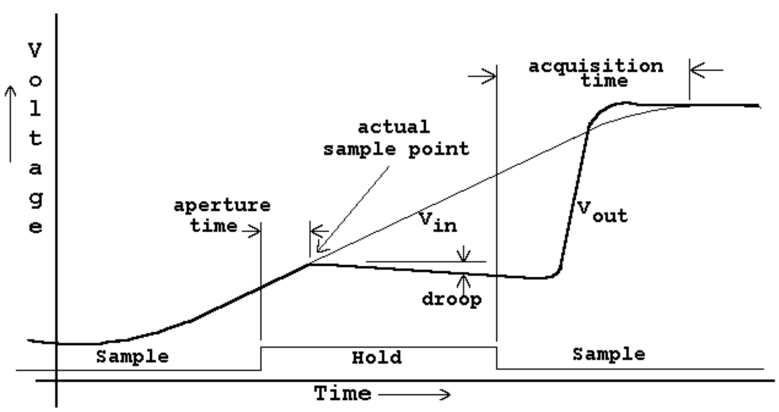


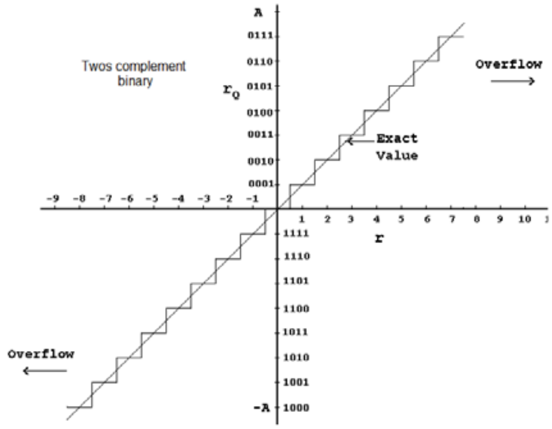
Figure 4.11
Left is the magnitude plot for the combined 3rd order analog filter and the 8th order digital filter. At right, the pass band and stop band.

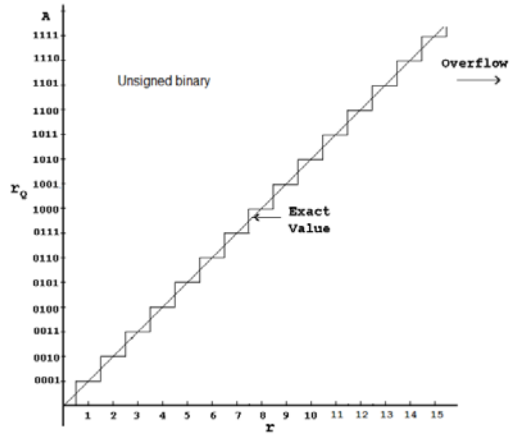
Sample and Hold





Quantization





Sample frequency in KHz	Sample period in µsec	Application
8	125	Telephone and wireless microphone
32	31.25	MiniDV such as camcorders
44.1	22.68	Audio CD
48	20.08	DVD sound, DAT (Digital Audio Tape)
88.2	11.34	Professional audio
96	10.42	DVD-audio
192	5.21	DVD-audio

A/D Conversion