

Second order systems

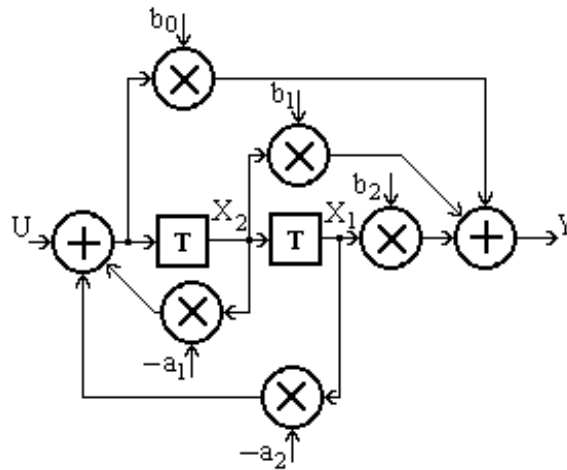
Transfer function

$$H(z) = \frac{y(z)}{u(z)} = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

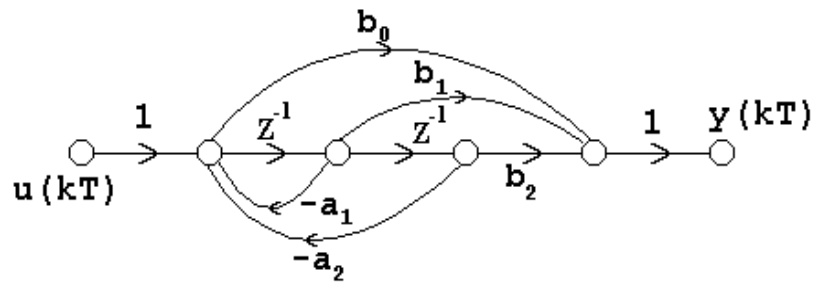
Difference equation

$$y_k = b_0 u_k + b_1 u_{k-1} + b_2 u_{k-2} - a_1 y_{k-1} - a_2 y_{k-2}$$

State Variable Representation Canonic Block diagram



Signal flow graph



State variable equations

$$x_1(k+1) = x_2$$

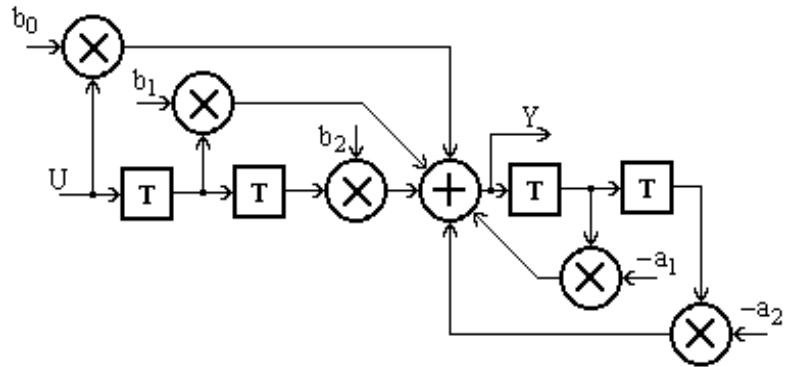
$$x_2(k+1) = -a_2 x_1 - a_1 x_2 + u_k \quad \Rightarrow \quad \overline{X(k+1)} = \overline{A} \cdot \overline{X(k)} + \overline{B} \cdot \overline{U(k)}$$

$$y_k = (b_2 - b_0 a_2) x_1 + (b_1 - b_0 a_1) x_2 + b_0 u_k \quad \Rightarrow \quad \overline{Y(k)} = \overline{C} \cdot \overline{X(k)} + \overline{D} \cdot \overline{U(k)}$$

where $\overline{X(k)} = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$ $\overline{Y(k)} = [y(k)]$ and

$$\overline{A} = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix} \quad \overline{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \overline{C} = [b_2 - b_0 a_2 \quad b_1 - b_0 a_1] \quad \overline{D} = [b_0]$$

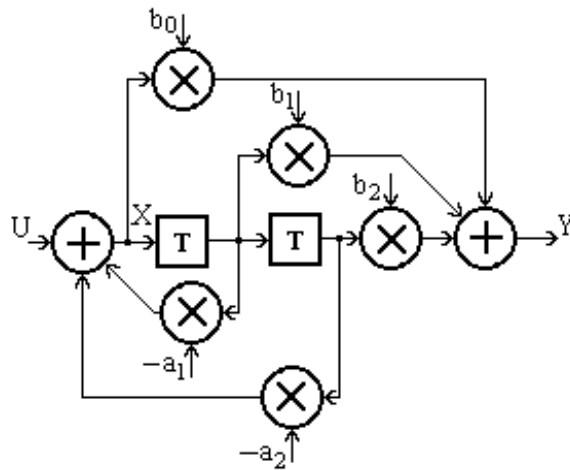
Direct form I
Block diagram



Direct form I equations

$$y_k = b_0 u_k + b_1 u_{k-1} + b_2 u_{k-2} - a_1 y_{k-1} - a_2 y_{k-2}$$

Direct form II
Block diagram



Direct form II equations

$$X(k) = U(k) - a_1 X(k-1) - a_2 X(k-2)$$

$$Y(k) = b_0 X(k) + b_1 X(k-1) + b_2 X(k-2)$$