Engr 123
Programming Assignment 3

Assigned: January 24, 2018
Due: February 5, 2018

Reminder: This is a programming project, and work on this assignment should be done individually. Assistance from other students is limited to questions about specific issues as noted in the syllabus.

For this problem we will calculate the area of a parabola using numerical integration. For a parabola which opens downward and passes through (0, 0) and (10, 0) the quation is given by

$$y = -kx(x-10) = -kx^2 + 10kx$$

Where k is a positive number greater than 0.

The value of k determines the height of the parabola. Figure 1 shows three parabolas with different values of k.

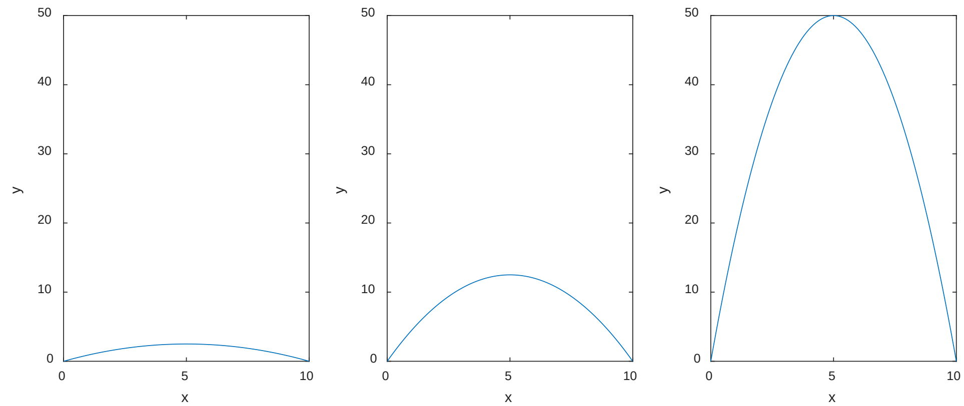

Figure 1
Three parabolas with k = 0.1, 0.5, and 2.0

If we evaluate the integral from 0 to 10 we can find the area under the parabola as:

$$area = \int_0^{10} (-kx^2 + 10kx)dx$$

This integral evaluates to:

$$area = [-kx^3/3 + 5kx^2]_0^{10} \;=\; -1000k/3 + 500k$$

We can also evaluate this integral using numerical methods.

Numerical Integration
There are several methods of doing numerical integration and in this problem we will explore two of them: Rectangular integration and trapezoidal integration.

Rectangular Integration with Backward Differences:
For rectangular integration we approximate the area under a curve by filling the curve with a sequence of narrow rectangles. Figure 2 shows an arbitrary function of time which has been sampled at regular intervals. If the width of a single sample is T, then the area

of a rectangle at point n is T times x_n. If we write $y = \int_0^a f(x) \cdot dt$ then we can approximate y as a succession of rectangular areas like this

$$y_n - y_{n-1} = T \cdot x_n$$
$$or \qquad (1)$$
$$y_n = y_{n-1} + T \cdot x_n$$

Thus if we have a sequence of values of f(x) for x going from 0 to say 5, we could find the value of the integral of f(x) for the interval 0 to 5 by using equation 1 successively.

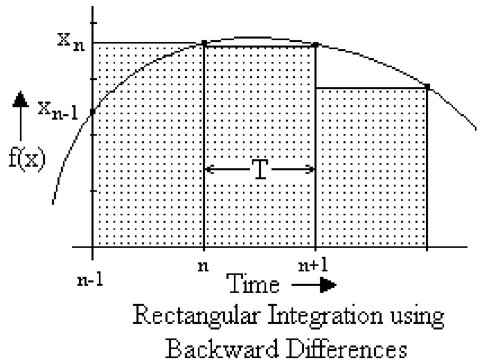

Figure 2
A function of x is sampled and divided into rectangles to estimate the area.

The pseudo code below explains how to evaluate the area under a semi-circle from –r to +r using 100 small rectangles:

```
incr = 0.1;
area = 0.0;
x = incr;
while(x <= 10)
   {area = area + incr*(-kx² +10kx);
    x = x + incr;
   }
print area
```

Trapezoidal Integration:
For trapezoidal integration we divide the function into trapezoids spaced at the sample intervals as shown in Figure 2. The area of a single trapezoid can be found by taking the average of the two ordinates and multiplying it by the width, T. The relevant equations are

$$y_n - y_{n-1} = T \cdot \frac{x_n + x_{n-1}}{2}$$
$$or \qquad (2)$$
$$y_n = y_{n-1} + T \cdot \frac{x_n + x_{n-1}}{2}$$

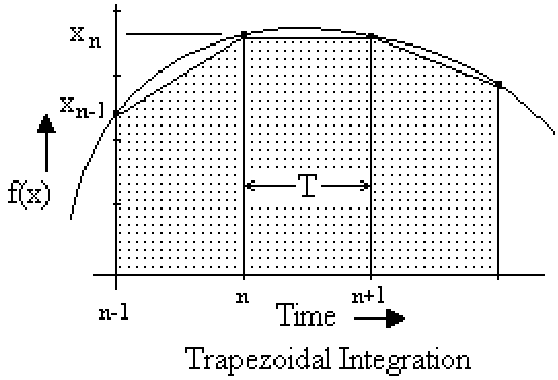

Figure 3

A function of x is sampled and divided into trapezoids to estimate the area.

For this assignment, you should integrate the parabola from 0 to 10 in 100 increments where *k* is input by the user. You should use both rectangular and trapezoidal integration and compare it to the true value for the integral over the same period. Your program should print something like the following:

```
Input a value for the k... 0.1
This program integrates f(x) = -kx(x-10)
over the interval 0 to 10 using rectangular and trapezoidal
integration.

The value of k is: 0.1
The true area is: 16.667

Using rectangular integration:
The area of the parabola with k = 0.1 is 16.665
Using trapezoidal integration:
The area of the parabola with k = 0.1 is 16.665
Press any key to continue . . .
```

You should calculate the true value by hand using a calculator.

Your program MUST be modular - that is, it should consist of a main program that is largely a sequence of function calls.

After you get your program running correctly, right click on the *project folder* and choose Send To → Compressed zip file. Rename the compressed zip file as Asn03XXX.zip where XXX are you three initials. Upload the renamed file to \\cecsfp01\users\everyone\engr123.