Where Does Euler's Identity Come From?
Consider the differential equation:
\[ \frac{dx}{dt} = jx \]

We can guess that the solution is \( x = e^{jt} \) so that \( \frac{dx}{dt} = je^{jt} \).

Putting these two back in the equation gives
\( je^{jt} = je^{jt} \) so \( x = e^{jt} \) is a solution to the equation.

Just for fun try a different solution say \( x = \cos(t) + j\sin(t) \)

We get \( \frac{dx}{dt} = -\sin(t) + j\cos(t) \)

Putting this back in the original equation we get:
\[ -\sin(t) + j\cos(t) = j(\cos(t) + j\sin(t)) = j\cos(t) - \sin(t) \]
So this too is a solution to the equation. But the equation is first order so it can have only one solution. Therefore
\( e^{jt} = \cos(t) + j\sin(t) \)

In a similar fashion you can show that
\( e^{-jt} = \cos(t) - j\sin(t) \) by using \( \frac{dx}{dt} = -jx \)

In general, Euler's identity is written as:
\( e^{\pm jx} = \cos(x) \pm j\sin(x) \)