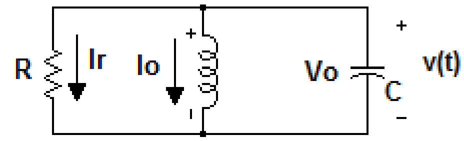
EE 210
Parallel RLC Notes

KCL



$$\frac{v}{R}+\frac{1}{L}\int_{-\infty}^{t} v\,dt + C\frac{dv}{dt}=0$$

or

$$\frac{d^2v}{dt^2}+\frac{1}{RC}\frac{dv}{dt}+\frac{v}{LC}=0$$

This is a second order linear differential equation with constant coefficients

Characteristic equation :

$$s^2+\frac{1}{RC}s+\frac{1}{LC}=0$$

Roots are:

$$s_1=-\frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2-\frac{1}{LC}} \text{ and } s_2=-\frac{1}{2RC}-\sqrt{\left(\frac{1}{2RC}\right)^2-\frac{1}{LC}}$$

Let $\alpha=\frac{1}{2RC}$ $= \textit{Damping factor}$

$\omega_0=\frac{1}{\sqrt{LC}}$ $= \textit{Resonant frequency in radians/sec}$

Then

$s_{1,2}=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}$ s_1 and s_2 are called *natural frequencies*

For $\alpha^2-\omega_0^2>0$ the system is *over damped.*

Solution is $i(t)=A_1e^{s_1t}+A_2e^{s_2t}$

For $\alpha^2-\omega_0^2=0$ the system is *critically damped.*

Solution is $i(t)=(A_1+A_2t)e^{-\alpha t}$

For $\alpha^2-\omega_0^2<0$ the system is *under damped.*

Solution is $i(t)=e^{-\alpha t}[B_1\cos(\omega_d t)+B_2\sin(\omega_d t)]$

where $\omega_d=\sqrt{\omega_0^2-\alpha^2}$