

**EE 210**  
**Series RLC Notes**

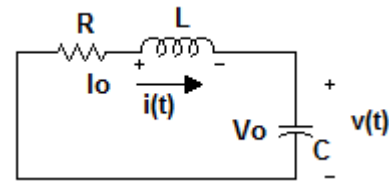
KVL

$$Ri + L \frac{di}{dt} + \frac{1}{C} \int_{-\infty}^t i dt = 0$$

or

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = 0$$

This is a second order linear differential equation with constant coefficients



Characteristic equation :

$$s^2 + \frac{R}{L} s + \frac{1}{LC} = 0$$

Roots are:

$$s_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \quad \text{and} \quad s_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Let  $\alpha = \frac{R}{2L} = \text{Damping factor}$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \text{Resonant frequency in radians/sec}$$

Then

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \quad s_1 \text{ and } s_2 \text{ are called } \textit{natural frequencies}$$

For  $\alpha^2 - \omega_0^2 > 0$  the system is *over damped*.

$$\text{Solution is } i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

For  $\alpha^2 - \omega_0^2 = 0$  the system is *critically damped*.

$$\text{Solution is } i(t) = (A_1 + A_2 t) e^{-\alpha t}$$

For  $\alpha^2 - \omega_0^2 < 0$  the system is *under damped*.

$$\text{Solution is } i(t) = e^{-\alpha t} [B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)]$$

$$\text{where } \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$