ECE307-11

Active Filter Circuits

Higher Order Op Amp Filter

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Higher Order Op Amp Filter

Cascading Identical Filter

To obtain a sharper transition between the pass-band and stop-band, we can add more identical filter in cascade.

One filter gives us 20dB/dec slope, two identical cascade filters give us 40 dB/dec slope transition.

n identical cascade low-pas filters give us from the corner frequency *n*20dB/dec slope

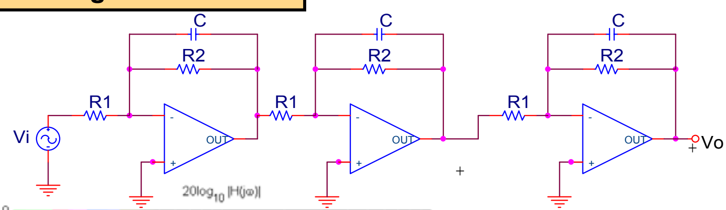
Transfer function

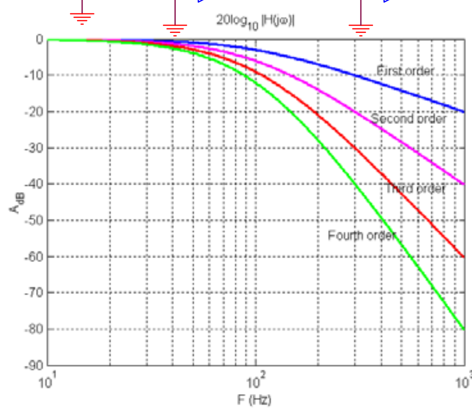
$$H(s) = \left(\frac{-\omega_c}{s+\omega_c}\right)\left(\frac{-\omega_c}{s+\omega_c}\right)\cdots\left(\frac{-\omega_c}{s+\omega_c}\right)$$

$$H(s) = \left(\frac{-\omega_c}{s+\omega_c}\right)^n$$

Higher Order Op Amp Filter

Cascading Identical Filter





New Cutoff frequency

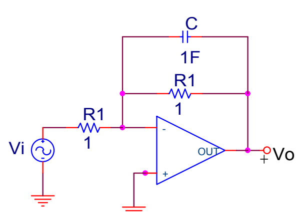
$$|H(j\omega)| = \left|\frac{-\omega_c}{j\omega_{cn} + \omega_c}\right|^n = \frac{1}{\sqrt{2}}$$

$$\omega_{cn} = \left(\sqrt{\sqrt[n]{2} - 1}\right)\omega_c$$

Higher Order Op Amp Filter

Example

- Design 4 order low-pass filter with cut-off frequency is 500Hz and a pass-band gain is 10. Use capacitor 1μF capacitors



From the cutoff frequency

$$\omega_{cn} = \left(\sqrt{\sqrt[n]{2} - 1}\right)\omega_c$$

$$\omega_c = \frac{\omega_{cn}}{\left(\sqrt{\sqrt[4]{2} - 1}\right)} = \frac{2\pi 500}{0.435} = 7222.39\ rad/s$$

$$\omega_c = \frac{1}{RC}$$

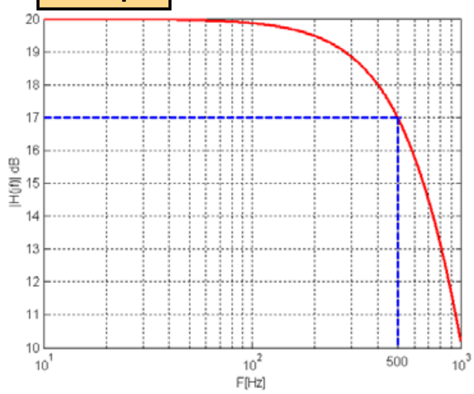
$$R = \frac{1}{\omega_c C} = \frac{1}{7222.39(10^{-6})} = 138.45\Omega$$

For gain specification, we need to change R_f

$$R_f = GR_2 = 10(138.45) = 1383.5\ \Omega$$

Higher Order Op Amp Filter

Example




```
>> f=10:1:1000;
>> w=2*pi*f;
>> wc=7222.39;
>> h=20*log10(10)-
80*log10(abs(1+j*w/wc));
>> semilogx(f,h)
>> grid on
>> xlabel(‘F[Hz]')
>> ylabel('|H(jf)| dB')
>>
```

$$A_{dB} = 20\log_{10}(10) + 20\log_{10}\left|1+\frac{j\omega}{\omega_c}\right|$$

Each filter Cutoff frequency

$\omega_c = 7222.39$ rad/s

Final Cutoff frequency

$\omega_{cn} = 2\pi 500$ rad/s

$$H(j\omega) = (-10)\left(\frac{-\omega_c}{j\omega+\omega_c}\right)^4 = (-10)\left(\frac{-1}{1+\frac{j\omega}{\omega_c}}\right)^4$$

Butterworth Filters

Butterworth Filters

A unity gain Butterworth Filter Transfer function

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\frac{\omega}{\omega_c})^{2n}}}$$

N is integer and denotes the order of the filter

The cutoff frequency is ω_c for all values of n
If n is large enough, the denominator is always close to unity when $\omega < \omega_c$
The exponent of $\omega/\ \omega_c$ is always even

We can set ω_c equal to 1 rad/s in the transfer function, then we use scaling transform

Butterworth Filters

Butterworth Filters

- We know that $|H(j\omega)|^2 = H(j\omega)H(-j\omega)$ $\quad |H(j\omega)|^2\,|_{j\omega=s} = H(s)H(-s)$

So we observe that $s^2 = -\omega^2$

Then
$$|H(j\omega)|^2 = \frac{1}{1+(\omega)^{2n}} = \frac{1}{1+(\omega^2)^n} = \frac{1}{1+(-s^2)^n} = \frac{1}{1+(-1^n)s^{2n}}$$

$$Hs)H(-s) = \frac{1}{1+(-1^n)s^{2n}}$$

Given value of a
- Find the roots of the polynomial $1+(-1^n)s^{2n} = 0$
- Assign the left-half plane roots to H(s) and the right-half plane roots to H(-s)
- Combine terms in the denominator of H(s) to form first and second order factors.

Butterworth Filters

Example

Find Butterworth filter transfer function for n=2 and n=3

For n=2 $\quad 1+(-1^2)s^4 = 0 \quad s^4 = -1 \quad s^4 = 1\angle 180^\circ$

Therefore the four roots

$$s_1 = 1\angle 45^\circ = \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} \qquad s_2 = 1\angle 135^\circ = -\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$$

$$s_1 = 1\angle 225^\circ = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} \qquad s_1 = 1\angle 315^\circ = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$$

Roots s2 and s3 in the left-half plane

$$H(s) = \frac{1}{(s+\frac{1}{\sqrt{2}}-\frac{j}{\sqrt{2}})(s+\frac{1}{\sqrt{2}}+\frac{j}{\sqrt{2}})} \qquad H(s) = \frac{1}{(s^2+\sqrt{2}s+1)}$$

Butterworth Filters

For n=3

$1+(-1^3)s^6 = 0$

$s^6 = 1$

$s^6 = 1\angle 360^\circ$

Therefore the four roots

$s_1 = 1\angle 0^\circ = 1$

$s_2 = 1\angle 60^\circ = \frac{1}{2} + j\frac{\sqrt{3}}{2}$

$s_3 = 1\angle 120^\circ = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$

$s_4 = 1\angle 180^\circ = -1$

$s_5 = 1\angle 240^\circ = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$

$s_6 = 1\angle 300^\circ = \frac{1}{2} - j\frac{\sqrt{3}}{2}$

Roots s_3, s_4 and s_5 in the left-half plane

$$Hs) = \frac{1}{(s+1)(s+\frac{1}{2} - j\frac{\sqrt{3}}{2})(s+\frac{1}{2}+j\frac{\sqrt{3}}{2})}$$

$$Hs) = \frac{1}{(s+1)(s^2+s+1)}$$

Butterworth Filters

Butterworth Polynomials (ω_c =1rad/s)

n	Nth order Butterworth polynomials
1	$(s+1)$
2	$(s^2+\sqrt{2}s+1)$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.765s+1)(s^2+1.848s+1)$
5	$(s+1)(s^2+0.618s+1)(s^2+1.618s+1)$
6	$(s^2+0.518s+1)(s^2+\sqrt{2}s+1)(s^2+1.932s+1)$

- A circuit can be scaled in both magnitude and frequency in simultaneously

$R' = k_m R$

$C' = \frac{C}{k_m k_f}$

Butterworth Filters

Butterworth Filter Circuit

- Fifth order Butterworth filter block diagram is shown in the following figure.

$$V_i \rightarrow \boxed{\frac{1}{s+1}} \rightarrow \boxed{\frac{1}{s^2+0.618s+1}} \rightarrow \boxed{\frac{1}{s^2+1.618s+1}} \rightarrow V_0$$

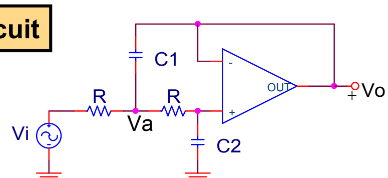
Each block indicates the transfer function. Buterworth filter cutoff frequency is ω_c =1rad/s

He have designed first order active filter in the previous classes. Now, we need to design second order active filter, which is

$$H(s) = \frac{1}{s^2+bs+1}$$

Butterworth Filters

Butterworth Low-Pass Filter Circuit



- Node equations

$$\frac{V_a - V_i}{R} + (V_a - V_0)sC_1 + \frac{(V_a - V_0)}{R} = 0$$

$$2 + RC_1V_a - (1 + RC_1s)V_0 = V_i$$

$$V_0sC_2 + \frac{(V_0 - V_a)}{R} = 0$$

$$-V_a + (1 + RC_2s)V_0 = 0$$

$$V_0 = \frac{Vi}{R^2C_1C_2s^2 + 2RC_2s + 1}$$

$$H(s) = \frac{V_0}{Vi} = \frac{\frac{1}{R^2C_1C_2}}{s^2 + \frac{2}{RC_1}s + \frac{1}{R^2C_1C_2}}$$

Butterworth Filters

Butterworth Filter Circuit

- Set R=1Ω, then

$$H(s)=\frac{V_0}{Vi}=\frac{\frac{1}{C_1C_2}}{s^2+\frac{2}{C_1}s+\frac{1}{C_1C_2}}$$

- Second order circuit in Butterworth filter

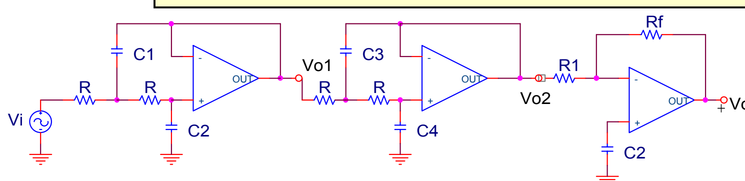
$$H(s)=\frac{1}{s^2+bs+1} \qquad b_1=\frac{2}{C_1} \qquad 1=\frac{1}{C_1C_2}$$

- Second order Butterworth filter with cutoff frequency of ω_c =1 rad/s and gain is 1.
- Use frequency scaling to calculate revised capacitor values for the wanted cutoff frequency
- Use magnitude scaling to provide more realistic component value

Butterworth Filters

Example

- Design a fourth order Butterworth low-pass filter with a cutoff frequency 500Hz and passband gain of 10.



- From the Butterworth polynomial table

$$(s^2+0.765s+1)(s^2+1.848s+1)$$

- Thus we need two cascade of two second order filters and inverting amplifier circuit for gain 10

- First stage of cascade

$$0.765=\frac{2}{C_1} \qquad 1=\frac{1}{C_1C_2} \qquad C_1=2.61\,F \qquad C_2=0.38\,F$$

Butterworth Filters

Example

Second stage of cascade

$(s^2 + 1.848s + 1)$

$1.848 = \frac{2}{C_3}$

$1 = \frac{1}{C_3 C_4}$

$C_3 = 1.08\ F$

$C_4 = 0.924\ F$

Frequency scaling factor f_c=500 Hz

$k_f = \frac{\omega_c}{1rad/s} = 2\pi 500 = 3141.6$

To have R=1KΩ, magnitude scaling factor

$k_m = 1000$

$R' = k_m R = 1\ K\Omega$

$C' = \frac{C}{k_m k_f}$

$C_1 = 831\ nF$

$C_2 = 121\ nF$

$C_3 = 344\ nF$

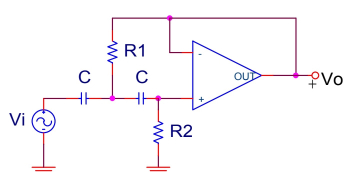
$C_4 = 294\ nF$

For inverting amplifier stage, let R_i=1K Ω

$R_f = 10R_i = 10\ K\Omega$

Butterworth Filters

Butterworth High-Pass Filter



Second order High-pass filter form

$$H(s) = \frac{s^2}{s^2 + b_1 s + 1}$$

The transfer function of the circuit

$$H(s) = \frac{V_0}{Vi} = \frac{s^2}{s^2 + \frac{2}{R_2 C}s + \frac{1}{R_1 R_2 C^2}}$$

Setting C=1F

$$H(s) = \frac{V_0}{Vi} = \frac{s^2}{s^2 + \frac{2}{R_2}s + \frac{1}{R_1 R_2}}$$

$b_1 = \frac{2}{R_2}$

$1 = \frac{1}{R_1 R_2}$

Example : Second order Butterworth filter

$(s^2 + \sqrt{2}s + 1)$

$R_2 = \frac{2}{\sqrt{2}} = 1.41\,\Omega$

$R_1 = \frac{1}{R_2} = 0.707\ \Omega$

Butterworth Filters

Example :

Second order Butterworth High-pass filter with cutoff frequency 1KHz gain 10. Use 0.1uF capacitor

From the Butterworth polynomial table $(s^2 + \sqrt{2}s + 1)$

We need a second order filters and an inverting amplifier circuit for gain 10

$b_1 = \frac{2}{R_2}$ $1 = \frac{1}{R_1 R_2}$ $R_2 = \frac{2}{\sqrt{2}} = 1.41\,\Omega$ $R_1 = \frac{1}{R_2} = 0.707\ \Omega$

Frequency scaling factor f_c=1000 Hz

$$k_f = \frac{\omega_c}{1\text{rad/s}} = 2\pi 1000 = 6283.2$$

To have C=0.1uF, magnitude scaling factor

$$k_m = \frac{C}{C' k_f} = \frac{1}{0.1(10^{-6})6283.2} = 1591.5$$

New R Values

$$R_{1n} = k_m R_1 = 1591.5\ (0.707) = 1125\ \Omega$$

$$R_{2n} = k_m R_2 = 1591.5\ (1.41) = 2244\ \Omega$$

Butterworth Filters

Example (Cont) :

For inverting amplifier stage, let R_i=1K Ω

$$R_f = 10R_i = 10\ K\Omega$$

Finally, we have

$$H(s) = \frac{V_0}{Vi} = \frac{s^2}{s^2 + \frac{2}{R_2 C}s + \frac{1}{R_1 R_2 C^2}}\left(-\frac{R_f}{R_i}\right)$$

$R_1 = 1125\ \Omega$ $R_i = 1\ K\Omega$

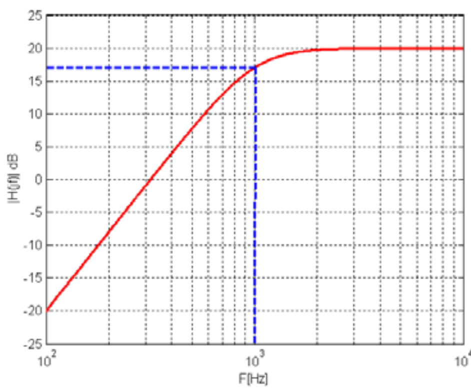
$R_2 = 2244\ \Omega$ $R_f = 10\ K\Omega$

$C = 0.1\ \mu F$

Butterworth Filters

Example (Cont) :

MatLab




```
>> R1=1125;
>> R2=2244;
>> C=0.1e-6;
>> Rf=10000;
>> Ri=1000;
>> f=100:1:10000;
>> w=2*pi*f;
>>
H=(j*w).^2./((j*w).^2+2./(R2
*C).*j*w+1./(R1*R2*C^2))*(-
Rf/Ri);
>> A=20*log10(abs(H));
>> semilogx(f,A);
>> grid on;
>> xlabel('F[Hz]')
>> ylabel('|H(jf)| dB')
```


Butterworth Filters

Orcad :