#1 2 b) FFFV, FFVF, FVFF, VFFF, VFFF

c) FFFF and VVVV

d) FFFF, FFFV, FVFF, VFFF

e) union: all in d) plus VVVV
intersection: FFFF

f) union is all listed in b) plus all listed in e)
intersection is the empty set - no events in common

#12 page 65

A = Visa  B = MasterCard

given P(A) = 0.5  P(B) = 0.3  P(A∩B) = 0.25

3 a) P(A∪B) = P(A) + P(B) - P(A∩B)
    = 0.5 + 0.3 - 0.25 = 0.55

b) P(A∪B') = 1 - P(A∪B) = 0.35

c) P(A∩B') = P(A) - P(A∩B) from diagram
    = 0.5 - 0.25 = 0.25

#18 page 65

15 bulbs total: 4 - 40W, 5 - 60W, 6 - 75W

P (at least 2) = 1 - P (1st is 75W) = 1 - \frac{6}{15} = \frac{9}{15} = \frac{3}{5} = 0.60
2. a) order is important, so we are interested in permutations of 3 of 8 objects

8!/(8-3)! = 336 ways if order is important

b) no order mentioned, line (\( \binom{30}{2} \)) = \( \text{choose}(30,2) = 59370 \)

c) \( \binom{8}{2} \binom{10}{2} \binom{12}{2} = \text{choose}(8,2) \cdot \text{choose}(10,2) \cdot \text{choose}(12,2) \)

= 83160

d) since part c) gives # ways to pick 2 of each and b) gives # ways to choose 6 bottles

Answer is \( \binom{8}{2} \binom{10}{2} \binom{12}{2} \) = 83160

= 0.14

e) there are three mutually exclusive ways for this to happen, since there are three categories of wine

Answer is \( \binom{8}{2} + \binom{10}{2} + \binom{12}{2} \)

= 0.002

using MATLAB

---

34. page 74

2. a) \( \binom{20}{6} = 38760 \) selections with all from day shift

Note: There are 20+15+10 = 45 workers altogether. If 20 are day shift, 25 are not.

\[ P(\text{all 6 day}) = \frac{\binom{20}{6} \binom{25}{6}} {\binom{45}{6}} \approx 0.00476 \]

b) there are three ways this could happen, so

\[ P(\text{all same}) = \frac{\binom{20}{6} \binom{25}{6} + \binom{15}{6} \binom{20}{6} + \binom{10}{6} \binom{15}{6}} {\binom{45}{6}} \]

\[ = \frac{\binom{20}{6} + \binom{15}{6} + \binom{10}{6}{2}} {\binom{45}{6}} \approx 0.0053 \]
c) This event is the complement of the event in part b).

Then \( P(=3 \text{ rep.}) = 1 - 0.0054 = 0.9946 \)

d) Need to define events for this

\[ A_1 = \text{day shift unrepresented} \quad P(A_1) = \left( \frac{25}{60} \right) / \left( \frac{35}{45} \right) = 0.2222 \]

\[ A_2 = \text{swing shift unrepresented} \quad P(A_2) = \left( \frac{25}{30} \right) / \left( \frac{45}{60} \right) = 0.3571 \]

\[ A_3 = \text{graveyard shift unrepresented} \quad P(A_3) = \left( \frac{25}{35} \right) / \left( \frac{45}{60} \right) = 0.3333 \]

\[ P(\text{at least 1 shift unrepresented}) = P(A_1 \cup A_2 \cup A_3) \]

\[ P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) \]

\[ P(A_1 \cap A_2 \cap A_3) = 0, \text{ since not all can be unrepresented} \]

\[ P(A_1 \cap A_2) = P(\text{all graveyard}) = \left( \frac{\binom{15}{15}}{\binom{45}{35}} \right) = 0.00002 \]

\[ P(A_1 \cap A_3) = P(\text{all swing}) = \left( \frac{\binom{15}{25}}{\binom{45}{35}} \right) = 0.00061 \]

\[ P(A_2 \cap A_3) = P(\text{all day}) = 0.00057 \text{ from part a) } \]

Thus \[ P(A_1 \cup A_2 \cup A_3) = 0.2222 + 0.3571 + 0.3333 \]

\[ = 0.9126 \quad \text{from part a) } \]

\[ = 0.2885 \]

45 page 83

3.

a) \[ P(A) = 0.106 + 0.141 + 0.200 = 0.447 \]

\[ P(C) = 0.215 + 0.300 + 0.185 + 0.020 = 0.540 \]

\[ P(C \cap C) = 0.020 \]

b) \[ P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{0.200}{0.540} = 0.370 \]

This is probability that person from ethnic group 3 has type A blood.

\[ P(C|A) = \frac{P(A \cap C)}{P(A)} = \frac{0.200}{0.447} = 0.447 \]

This is probability that person with type A blood is from ethnic group 3.
3. Define event D = 2 individual from ethnic group A.

\[P(D) = 0.082 + 0.106 + 0.004 = 0.292\]

We want \[P(D|B') = \frac{P(D \cap B')}{P(B')}\]

\[P(D \cap B') = 0.082 + 0.004 = 0.196\]

\[P(B') = 1 - P(B) = 1 - [0.082 + 0.106 + 0.004] = 0.796\]

So \[P(D|B') = \frac{0.196}{0.796} < 0.25\]

#61 page 85

**b.** batch s. & samples

3. Want to find \[P(0 \text{ def. in } S | \text{ def. in } B), P(1 \text{ def. in } S | 0 \text{ def. in } S), P(2 \text{ def. in } S | 0 \text{ def. in } S)\]

\[P(0 \text{ def. in } S) = P(0 \text{ def. in } S | 0 \text{ def. in } B) P(0 \text{ def. in } B) + P(0 \text{ def. in } S | 1 \text{ def. in } B) P(1 \text{ def. in } B) + P(0 \text{ def. in } S | 2 \text{ def. in } B) P(2 \text{ def. in } B)\]

We are given:

\[P(0 \text{ def. in } B) = 0.500\]

\[P(1 \text{ def. in } B) = 0.300\]

\[P(2 \text{ def. in } B) = 0.200\]

\[P(0 \text{ def. in } S | 0 \text{ def. in } B) = 1.000\]

\[P(0 \text{ def. in } S | 1 \text{ def. in } B) = \frac{\binom{2}{0}}{\binom{2}{1}} \cdot 0.800\]

\[P(0 \text{ def. in } S | 2 \text{ def. in } B) = \frac{\binom{2}{0}}{\binom{2}{2}} \cdot 0.622\]

Thus \[P(0 \text{ def. in } S) = (1)(0.500) + (0.800)(0.300) + (0.622)(0.200) = 0.864\]

\[P(0 \text{ def. in } S | 0 \text{ def. in } B) = P(0 \text{ def. in } S | 0 \text{ def. in } B) / P(0 \text{ def. in } S) = 0.5 / 0.864 = 0.578\]

\[P(1 \text{ def. in } S | 0 \text{ def. in } S) = P(1 \text{ def. in } S | 0 \text{ def. in } S) / P(0 \text{ def. in } S) = 0.8(0.3) / 0.864 = 0.278\]
Since \( P(\text{day in b. } \cap \text{no def. in s.}) = P(\text{def. in s.}) / P(\text{day in b.}) P(\text{no def. in s.}) \),

\[
P(\text{2 def. in b. } | \text{def. in s.}) = P(\text{2 def. in b. } \cap \text{def. in s.}) / P(\text{def. in s.})
\]

\[
= (0.200)(0.622) / 0.8644 = 0.144
\]

b) Want to find \( P(\text{od. in b. } | \text{def. in s.}) \)

\[
P(\text{1 def. in b. } | \text{def. in s.})
\]

\[
P(\text{2 def. in b. } | \text{def. in s.})
\]

\[
P(\text{od. in b. } | \text{def. in s.}) = P(\text{od. in b. } \cap \text{no def. in s.}) / P(\text{def. in s.})
\]

\[
P(\text{1 def. in s. } | \text{def. in b.}) = 0
\]

\[
P(\text{1 def. in s. } | \text{2 def. in b.}) = \frac{\binom{9}{1}}{\binom{12}{2}} = 0.2
\]

\[
P(\text{1 def. in s. } | \text{2 def. in b.}) = \frac{\binom{3}{1} \cdot \binom{7}{1}}{\binom{12}{2}} = 0.3546
\]

Thus \( P(\text{1 def. in s.}) = 0.5 + 0.2(0.3) + 0.3546(0.2) = 0.1312 \)

\[
P(\text{od. in b. } \cap \text{1 def. in s.}) = 0
\]

\[
P(\text{1 def. in b. } \cap \text{1 def. in s.}) = P(\text{1 def. in s. } \cap \text{1 def. in b.})
\]

\[
= P(\text{1 def. in s. } | \text{1 def. in b.}) P(\text{1 def. in s.})
\]

\[
= 0.2(0.3) = 0.06
\]

Similarly \( P(\text{2 def. in b. } \cap \text{1 def. in s.}) = 0.3546(0.2) = 0.071
\]

* And \( P(\text{od. in b. } | \text{1 def. in s.}) = 0
\]

\[
P(\text{1 def. in b. } | \text{1 def. in s.}) = 0.06 / 0.1312 = 0.457
\]

\[
P(\text{2 def. in b. } | \text{1 def. in s.}) = 0.071 / 0.1312 = 0.542
\]
# 69. p. 90

\[ P(A) = 0.4 \] \[ P(B) = 0.7 \] \[ A \text{ and } B \text{ are independent events} \]

\[ \frac{P(B')}{P(A')} = P(B \cap A) / P(A) \]

\[ \text{Note: } P(A' \cap B') = P(AUB) \text{ since } A' \cap B' = (AUB)' \]

See Venn Diagram

Thus, \[ P(A' \cap B') = 1 - \left( P(A) + P(B) - P(A \cap B) \right) = 1 - \left( P(A) + P(B) - P(A)P(B) \right) \]

since \( A \) and \( B \) are independent

\[ P(A' \cap B') = 1 - \left( P(A) - P(A)P(B) \right) = P(B') - P(A)P(B) = [1 - P(A)] P(B) \]

Also see paragraph following equation 3.7 in text

Thus \[ P(C | B') = P(C \cap B') / P(B') = \frac{P(C) - P(C \cap B)}{1 - P(B)} = 1 - 0.7 = 0.3 \]

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.7 - 0.4 \times 0.7 = 0.82 \]

\[ b) \quad P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.7 - 0.4 \times 0.7 = 0.82 \]

\[ c) \quad \text{Want to find } P[A \cap B' | (A \cup B)] = \frac{P[A \cap B'] P(A \cup B)}{P(A \cup B)} \]

See Venn Diagram below

\[ P(A \cap B') = P(A) - P(A \cap B) = P(A) - P(A)P(B) = P(A) \left[ 1 - P(B) \right] = P(A)P(B') \]

From part b) \[ P(A \cup B) = 0.82 \]

Thus \[ P[A \cap B' | (A \cup B)] = \frac{P(A \cap B') P(A \cup B)}{P(A \cup B)} = \frac{0.4(1 - 0.7)}{0.82} = 0.146 \]
From the system diagram:

\[ P(\text{system works}) = P(\{1 \text{ works} \} \cup \{2 \text{ works} \}) \cup \{3 \text{ works} \} \cap \{4 \text{ works} \} \]
\[ = P(\{1 \text{ works} \} \cup \{2 \text{ works} \}) + P(\{3 \text{ works} \} \cap \{4 \text{ works} \}) - P(\{1 \text{ works} \} \cup \{2 \text{ works} \}) \cap \{3 \text{ works} \} \cap \{4 \text{ works} \} \]

Since all components work independently of one another, and P(component works) = 0.9,

\[ P(\{3 \text{ works} \} \cap \{4 \text{ works} \}) = 0.9 \times 0.9 = 0.81 \]
\[ P(\{1 \text{ works} \} \cup \{2 \text{ works} \}) = 0.9 + 0.9 - 0.9 \times 0.9 = 0.99 \]

So, P(system works) = 0.99 + 0.81 - (0.99)(0.81) = 0.998