Final Exam:

MATLAB

Arrays, operations (arithmetic, relational)
programming structures: for/while loops, if...elseif...else...end
standard mathematical functions in MATLAB, scientific notation
creating MATLAB functions, for example
functions that sum a series, functions that create/process arrays,
functions that calculate (polynomials, interpolants, iterations, etc)
simulating cellular automata (nothing specific on Wolfram’s coding system)
graphics: 2-D plots, of functions and parametric curves, images of arrays via
imagesc/image, colormaps
images of function data via pcolor. 3-D plots via surf, domains in polar coordinates
preparing arrays for plotting using meshgrid, plotting in other coordinate systems.
Plotting curves in space.

Math
Iteration $x_{n+1} = f(x_n)$, fixed points and convergence/divergence, stability/instability
linear convergence and the derivative $f'(x^*)$, checking for linear convergence experimentally
Newton’s method for finding roots of functions, quadratic convergence
Iteration/recurrence relations in general - developing a sequence of numbers, arrays, etc
according to a rule
Interpolation: Formulating equations, calculating the interpolant,
plotting the interpolant, the data, and the error (if applicable). Standard polynomial
interpolation, interpolation with special families (e.g. odd powers only, sines and cosines,
etc)

Least squares:
The least-squares solution of a linear system
Least squares fit of (x,y) data
Least square approximation of a function by sampling data
Plotting the least squares data fit along with the data, plotting the error (relative to
the datapoints or to the underlying function if there is one)
How to simulate a discrete probability distribution
1 with prob p, 0 with prob 1-p : u=(rand<p) or u=(rand(1,N)<p)
A random matrix of 1’s and 0’s with probability p of 1
A distribution: 1 with prob $p_1$, 2 with prob $p_2$, ..., n with prob $p_n$
Displaying the results of a simulation using $N=\text{hist}(Y,X)$ where X specified "bin centers"
Simulating a random walk with steps of ±1/-1 or random size in [-1,1], 2*(rand>.5)-1 or 2*rand-1 respectively; keeping track of stuff (what was the max or min, where did you wind up, how many times did you hit zero, etc)
FINAL EXAM – MATLAB Programming

NAME ___________________________________________

SECTION NUMBER _______________________________

CAMPUS MAILBOX NUMBER _______________________

EMAIL ADDRESS ________________________________@rose-hulman.edu

Problem 1 _________________ / 40

Problem 2 _________________ / 60

Total _________________ / 100

When you are done, we are going to have you collect all of your files into a folder called lastname_firstname, and copy/paste that folder to DFS. You may wish to create that folder now, and do all your work there from the beginning.
Problem 1 (40 points)

In this problem we are going to draw butterflies with Matlab. The final picture will look like this:

First, you will make a function that returns the \(x\) and \(y\) values that can be used to plot a single butterfly. (We give you the equations, see below.) Then you will call it the first time and plot a blue butterfly, the second time and plot a red butterfly, and then the third time to plot the green butterfly.

(a) Write a function to generate the \(x\) and \(y\) values for a single butterfly using a for loop. The equations for a butterfly were discovered by Temple H. Fey\(^1\) in 1989:

\[
\begin{align*}
    f &= e^{\cos t} - 2 \cos(4t) - \left(\sin\left(\frac{t}{12}\right)\right)^5 \\
    x &= x_0 + s \cdot f \cdot \sin t \\
    y &= y_0 + s \cdot f \cdot \cos t
\end{align*}
\]

Here \(t\) goes from 0 to 40\(\pi\), \(x_0\) and \(y_0\) are the location of the butterfly center, and \(s\) scales the height of the butterfly. Notice that these are algebraic equations, not differential equations. DO NOT USE EULER!

---

The function statement should look like this:

\[
\text{function } [x, y] = \text{butterfly\_function}(t_{\text{min}}, t_{\text{max}}, dt, s, xo, yo);
\]

where \( dt \) is the sampling step.

(b) Write a main program called \textit{problem1.m} that calls your function to generate the curve for the blue butterfly. It has \( t_{\text{min}}=0, t_{\text{max}}=40\pi, dt=0.001, s=0.1, xo=1, yo=1 \). In the main program, plot the butterfly with a blue line and add the commands

\[
\text{axis}([0 4 0 4]) \quad \text{axis equal} \quad \text{hold on}
\]

(This will make your plot have the same axes as the one on the previous page.)

(c) Add code to your main program to create a red butterfly with \( s=0.2, xo=3, yo=3 \), and a green butterfly with \( s=0.4, xo=2.5, yo=1 \). (That is, call your butterfly function with the new parameters and add additional plot commands to your main program. Even if you cannot get the butterfly function working correctly, show how you would have added this code to the main program.)

Instructions on posting these files to DFS are found on the last page.
Problem 2 (60 points)

A dynamic system is described by the following set of equations in which $t$, time, is the independent variable.

\[ \frac{dx}{dt} = y \]
\[ \frac{dy}{dt} = 0.04y - 1.2x \]
\[ \frac{dz}{dt} = 0.25 \]

The initial conditions are: $x(0) = 2.0$, $y(0) = 0.0$, and $z(0) = 0.0$.

(a) Using Euler’s method, write a program called `problem2.m` which will calculate the values of $x$, $y$ and $z$ for times from 0 to 15.

(b) Your program must then use the Matlab function, `plot3`, to draw a curve which represents the system’s time history in the $xyz$ space. (Use the Matlab help or documentation to learn more about `plot3`.) See below.

(c) Your program must calculate values of $x$, $y$ and $z$ at time 15 which are accurate to two decimal places, i.e. 0.01. Print the results to the command window with the following format:

```matlab
fprintf(’At time 15.0, x = %6.2f y = %6.2f z = %6.2f\n’, x(end),y(end),z(end))
```

(Note: `x(end)` prints the last entry in a vector.)

Instructions on posting your files to DFS are found on the last page.
Partner/Team Evaluation for ME123 Projects

Project 1: (Optimize Robot Object Pickup and Binning, simulation)

Name of Partner: ________________________________________________

Portion of the work on the project completed by you _____ % by your partner _____%

Comments:

Project 2: (Pick up and bin objects by color with actual robot)

Name of Partner: ________________________________________________

Portion of the work on the project completed by you _____ % by your partner _____%

Comments:

Project 3: (Draw logo with a robot)

Name of Partner: ________________________________________________

Portion of the work on the project completed by you _____ % by your partner _____%

Comments:
When you are done, create a folder on your laptop called lastname_firstname. Put all of your m-files in this folder. (There should be at least three files: butterfly_function.m, problem1.m, and problem2.m. If you created other functions be sure to include them as well.) Before you submit, run your code in the new folder to make sure all the necessary functions are included there.

Post your folder to the correct location on DFS:

1. Double-click on “Documents” on your laptop.
2. Double-click on “DFS Root” on the left column.
4. Double-click on Academic Affairs.
5. Double-click on ME.
6. Double-click on ME123.
7. Double-click on Exams.
8. Double-click on the folder with your section number.
9. Copy and paste your lastname_firstname folder to this location.
FINAL EXAM – MATLAB Programming

NAME ___________________________________________

SECTION NUMBER _______________________________

CAMPUS MAILBOX NUMBER _______________________

EMAIL ADDRESS ________________________________@rose-hulman.edu

Problem 1 _________________ / 60

Problem 2 _________________ / 20

Problem 3 _________________ / 20

Total _________________ / 100
**Problem 1 (60 points)**

A simple robotic arm can be modeled as a pendulum, like in the picture to the right. Let $\theta$ be the angular displacement of the pendulum counterclockwise from the downward position, and denote the angular velocity by $\omega$. There is damping in the pivot, and a force $F$ is applied horizontally to the lumped mass at the end of the pendulum.

The equations governing the behavior of a particular pendulum are as follows:

\[
\frac{d\omega}{dt} = -0.4 \omega - 20 \sin(\theta) + F \cos(\theta)
\]

\[
\frac{d\theta}{dt} = \omega
\]

where the units for $\theta$ and $\omega$ are rad and rad/s, respectively, and time $t$ is in seconds. Suppose the applied force $F$ is such that

- $F = 0$ when $\omega \geq 0$
- $F = 2$ when $\omega < 0$.

The initial conditions for the pendulum are

- $\omega(0) = 4$ rad/s
- $\theta(0) = 0$ rad.

Complete the following:

1. (40 points) Use MATLAB to plot the pendulum’s angular displacement $\theta$ as a function of time $t$, starting at $t = 0$ s and ending at $t = 4$ s. Use a time step $\Delta t = 0.01$ s. Be sure to properly label your axes and to include a title.

2. (20 points) Determine the values of $\theta$ and $\omega$ at $t = 1.737$ s. Note that your time vector does not have this exact value of time in it. Print these values to the screen using the following format:

   Angular displacement at $t = 1.737$ s is X.XXX rad.
   Angular velocity at $t = 1.737$ s is X.XXX rad/s.

Name your m-file `lastnameFirstname_problem1.m` (all lower case). Instructions on posting this file to DFS are found on Page 4.
Problem 2 (20 points)

In this problem, you will be asked to write a MATLAB function which determines the volume of fluid, with a variable height $h$, contained in a feeding trough with a geometry depicted below:

The radius of the semi-circular base is 0.75 m. Your function should perform the following tasks in an efficient way:

- receive only one input variable $h$ from a main calling code.
- internally define a feeding trough height $H$ of 3 m, a trough length $L$ of 5 m, and a semi-circular radius $R$ of 0.75 m.
- check if the input is physically meaningful, i.e. fits within the physical constraints of the feeding trough. If not, return a zero value for the output volume and print a warning message to the user:
  
  Error message: Your input fluid level is outside the physical range!

- return the fluid volume for the given fluid level $h$, in cubic meters, for the two cases:
  
  Fluid level is in the semi-circular base:
  
  $$ V = R^2 L \left[ \theta - \frac{\sin(2\theta)}{2} \right] $$
  
  where $\theta$ is in radians and defined by
  
  $$ \theta = \cos^{-1} \left( 1 - \frac{h}{R} \right) $$
  
  Fluid level is in the rectangular top:
  
  $$ V = 2RL\left(h-R\right) + \frac{\pi R^2}{2}L $$

- print out the result in the following format:
  
  The total fluid volume is $x.xx\times e+xxx$ cubic meters at a fluid level of $x.xx e+xxx$ meters.

Name your function $lastname\_firstname\_problem2.m$ (all lower case). Instructions on posting this file to DFS are found on Page 4.
Problem 3 (20 points)

Earth’s atmosphere can be divided into several layers. The tropopause, the boundary between the troposphere and the stratosphere, begins at an altitude of 11.0 km (11,000m) and extends to an altitude of 25.0 km (25,000m). While the temperature throughout this layer is approximately constant at 216.66 K, the following equation describes how the air density, $\rho$, changes with altitude, $h$:

$$\rho = \rho_0 \exp \left[\frac{g}{RT} \left( h - h_0 \right) \right]$$

where $g$ is the gravitational acceleration (9.81 m/s$^2$), $R$ is the gas constant for air (287 J/(kg K)), and $T$ is the temperature (216.66 K). The variables $h_0$ and $\rho_0$ are constant reference values of 11000 (meters) and 0.3648 (kg/m$^3$), respectively.

Your task is to write a MATLAB program to print a chart of altitude values from 11,000m to 25,000m along with the corresponding values of air density TWO DIFFERENT WAYS. Specifically, your program should print the chart using a “for” loop and then print it again using a “while” loop. In each case the chart should look the same. It should look like the following:

<table>
<thead>
<tr>
<th>Altitude (m)</th>
<th>Density (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11000</td>
<td>0.3648</td>
</tr>
<tr>
<td>12000</td>
<td>0.3116</td>
</tr>
<tr>
<td>13000</td>
<td>0.2661</td>
</tr>
<tr>
<td>14000</td>
<td>0.2273</td>
</tr>
<tr>
<td>15000</td>
<td>0.1941</td>
</tr>
<tr>
<td>16000</td>
<td>0.1658</td>
</tr>
<tr>
<td>17000</td>
<td>0.1416</td>
</tr>
<tr>
<td>18000</td>
<td>0.1209</td>
</tr>
<tr>
<td>19000</td>
<td>0.1033</td>
</tr>
<tr>
<td>20000</td>
<td>0.0882</td>
</tr>
<tr>
<td>21000</td>
<td>0.0753</td>
</tr>
<tr>
<td>22000</td>
<td>0.0643</td>
</tr>
<tr>
<td>23000</td>
<td>0.0549</td>
</tr>
<tr>
<td>24000</td>
<td>0.0469</td>
</tr>
<tr>
<td>25000</td>
<td>0.0401</td>
</tr>
</tbody>
</table>

Name your code for this problem lastnameFirstname_problem3.m (all lower case).

When you are done, post your m-file answer to the correct folder on DFS:

1. Double-click on “Documents” on your laptop.
2. Double-click on “DFS Root” on the left column.
3. Double-click on Academic Affairs.
4. Double-click on ME.
5. Double-click on ME123.
7. Double-click on the folder with your section number.
8. Copy and paste all your m-files to this folder.
EXAM 4 – Matlab Programming

NAME ___________________________________________

SECTION NUMBER _______________________________

CAMPUS MAILBOX NUMBER _______________________

EMAIL ADDRESS ________________________________@rose-hulman.edu

Problem 1 _________________/60
Problem 2 _________________/20
Problem 3 _________________/20
Total _________________/100
Problem 1 (60 points)

A shock absorber system can be modeled as a mass, a spring and a damper as in the picture below. The mass can only move back and forth in the x-direction. Its displaced position will be designated by \( x \), and its velocity by \( v \).

An engineer has derived the following equations to describe the behavior of the system. (Assume that we are using SI units, with kilograms, meters, and seconds.)

\[
\frac{d v}{d t} = -2.400 v - 4.000 x \\
\frac{d x}{d t} = v
\]

At time zero, an impulse loading is supplied to the system. The effect is to give the mass an instantaneous velocity in the positive x-direction. The initial conditions may then be written as follows.

\[
v(0) = 2.000 \quad x(0) = 0.000
\]

The resulting motion of the mass is an exponentially decaying sine curve. The mass will move to the right and reach a maximum position \( x_{\text{max}} \) at time \( t_{\text{max}} \). The mass will then move back to the left, passing through the starting point and reach a minimum (maximum negative valued) position \( x_{\text{min}} \) at time \( t_{\text{min}} \). Further oscillations of decreasing amplitude will follow.

1. (40 points) Use Matlab to plot the value of displaced position \( x \) as a function of time starting at time 0 and ending at a time of 5.
2. (20 points) The program should print out the values of \( x_{\text{max}} \), \( t_{\text{max}} \), \( x_{\text{min}} \) and \( t_{\text{min}} \) with statements using the following format.

   Maximum displacement of X.XXX meters occurs at time X.XXX seconds.

   Minimum displacement of -X.XXX meters occurs at time X.XXX seconds.

To get full credit in Part 2, every digit reported must be correct. In other words, your program must be accurate enough to calculate values which will correctly round to three decimal places.

Name your m-file \( \text{lastname\_firstname\_problem1.m} \) Instructions on posting this file to DFS are found on Page 4.
Problem 2 (20 points)

The pressure vessel shown at right is made up of a cylinder with a radius, $R$, of 0.75 m and a length, $L$, of 2.0 m with hemispherical ends each having the same radius, $R$, of 0.75 m.

You must write a function that will calculate the volume of water in the cylinder given the water level in the tank as input. The water level is given by the variable $h$ measured from the very bottom of the tank.

If the water level does not exceed the level of the lower hemisphere, then the volume of water, $V$, is given by the expression:

$$ V = \pi h^2 R - \frac{\pi}{3} h^3 $$

If the water level is somewhere in the straight section in the middle, then the volume is given by the following expression:

$$ V = \frac{2}{3} \pi R^3 + \pi R^2 (h - R) $$

If the water level is in the upper hemisphere, then the volume is given by the following expression:

$$ V = \frac{2}{3} \pi R^3 + \pi R^2 L + \pi R^2 (h - L - R) - \frac{\pi}{3} (h - L - R)^3 $$

Your function should have one input for water level ($h$) and one output for water volume ($V$). Name your function `lastnameFirstname_problem2.m` (all lower case). Upload only this function to the DFS drive, following the instructions on Page 4.
Problem 3 (20 points)

Suppose we want to make a contour plot of the function

\[ z = \sin(x) \sin(y) \]

for values of \( x \) from 0 to \( 2\pi \) and values of \( y \) from 0 to \( 2\pi \). The command \texttt{contour} will make a contour plot, in which each colored line represents a level, or height, \( z \) above or below the \( x-y \) plane. To use the \texttt{contour} function, the values for the \( x \)-axis need to be in an \( x \) vector which has \( m \) elements, and the values for the \( y \)-axis need to be in a separate \( y \) vector which has \( n \) elements. The array \( z \) has \( m \) rows and \( n \) columns.

Write a program to put the correct values in the vectors and the array and make a contour plot with 7 contour levels. Please use \texttt{help} and \texttt{doc} as needed. Choose \( m \) and \( n \) (they can even be equal), so that the curves are smooth. The plot should look similar the figure below, but smoother and with more contour lines.

Name your code for this problem \texttt{lastname_firstname_problem3.m} (all lower case). When you are done, post your m-file answer to the correct folder on DFS:

1. Double-click on “My Network Places”. If it is not on your desktop, look in your start menu.
3. Log in with your email address and password.
4. Double-click on Academic Affairs.
5. Double-click on ME.
6. Double-click on ME123.
7. Double-click on Exams.
8. Double-click on the folder with your section number.
9. Copy and paste all your m-files to this folder.
EXAM 1 – WRITTEN PORTION

NAME ___________________________________________
SECTION NUMBER _______________________________
CAMPUS MAILBOX NUMBER _____________________
EMAIL ADDRESS ________________________________@rose-hulman.edu

<table>
<thead>
<tr>
<th>Multiple Choice</th>
<th>/40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coding Problem</td>
<td>/60</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>/100</td>
</tr>
</tbody>
</table>
ALL OF THESE PROBLEMS HAVE EQUAL WEIGHT

USE MATLAB SYNTAX FOR ALL PROGRAMS AND COMMANDS YOU WRITE

**Problem 1:**
Write a short program using a `for…end` loop to find the sum of the squares of the numbers from 5 to 10. Assign the answer to a variable called `total`. You do not need to print the answers with `fprintf`.

**Problem 2:**
Write a short program using a `for…end` loop to find the square roots of the numbers 0, 10, 20...200. You do not need to print the answers with `fprintf`.

**Problem 3:**
Write a short program using a `for…end` loop to put the angles 0, 10, 20...90 into an array called `angle_array`. The array should have 10 rows and 1 column. You do not need to print the answers with `fprintf`.

**Problem 4:**
Circle ALL of the file names that would be appropriate to use in Matlab. Appropriate files names will run and not result in errors.
- a) Lab 1 task 2.m
- b) 1sttaskforlab1.m
- c) plot.m
- d) MyName_4.m
- e) plot_lab1.m
- f) lecture-2-ex-3.m
Problem 5:
In MATLAB, an array $A$ has been defined as follows:

$$
A = \begin{bmatrix}
1 & 3 & 5 \\
9 & 11 & 13 \\
2 & 4 & 6
\end{bmatrix}
$$

For the MATLAB command $y = A(3,1)$, what does MATLAB output for $y$?

Problem 6:
Using the same matrix $A$ from problem 5, we now execute the following MATLAB commands:

```matlab
B = A';
z = B(2,3)
```

What does MATLAB output for $z$?

Problem 7:
For the following mathematical expression, write the MATLAB code for $y$. Use the MATLAB command for $\pi$ and not 3.14159.

$$
y = (x + \pi)^2
$$
Problem 8:
When we run the MATLAB program shown below, an error statement appears in the Command window.

Here’s the program:

```matlab
% exampleProgram.m
% This program produces an mx1 array of angles named
% degrees and an mx1 array of the sines of the angles
% named sineAngle.

for angle = 0 : pi/10 : 2*pi
    degrees(row,1)   = angle*180/pi;
    sineAngle(row,1) = sin(angle);
    row = row + 1;
end

% last line
```

Here’s the error statement:

```matlab
??? Undefined function or variable 'row'.

Error in ==> exampleProgram at 8
degrees(row,1)   = angle*180/pi;
```

Modify the program to correct the error.
Problem 9:
Consider the following piece of code:

```matlab
fred = 1;
for index = 1:3:8
    fred = fred*index;
end
fred
```

What result will MATLAB produce in the Command window? (Circle one.)

a) fred = 1  

b) fred = 7  

c) fred = 21  

d) fred = 28  

e) fred = 280  

f) Other: ____________________

Problem 10:
Consider the following piece of code:

```matlab
index = 0;
for count = 3:-1:1
    index = index + 1;
    A(1,index) = count;
end
A
```

What is A after the code is executed? (Circle one.)

a) A = [1 2 3]  

b) A = [3 2 1]  

c) A = [1
        2
        3]  

d) A = [3
        2
        1]  


f) Other: ____________________
EXAM 1 – COMPUTER PORTION

Put all of your code in one m-file and name it: lastname_firstname.m (all lower case).

Include your name, section number, and CM number in the header section of your code. There should be no output other than what is asked for.

Problem (60 pts)
Download the Excel spreadsheet named “rate.xls” from the course web page at http://www.rose-hulman.edu/ME123/courseware.shtml. The file contains one column of annual investment rates (as percentages) for the years 1993 through 2007 (15 years total).

Write a MATLAB code to:

a) Read in the rate data stored in the Excel spreadsheet. Print the year and the rate for that year to a text file lastname_firstname1.txt, where lastname and firstname should be replaced by your last and first names, respectively. Do not forget to use the fclose command in your code. The first 3 lines of the text file should be:

<table>
<thead>
<tr>
<th>Year</th>
<th>Rate (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993</td>
<td>11.28</td>
</tr>
<tr>
<td>1994</td>
<td>-0.06</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
</tr>
</tbody>
</table>

b) Assume you started with $1000 in an investment account at the end of 1992. Use the annual investment rates provided, determine the balance in the account at the end of each year from 1992 through 2007, with no withdrawal. Print your end-of-year balances to the screen with the first 3 lines displaying the following format:

<table>
<thead>
<tr>
<th>Year</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>$1000.00</td>
</tr>
<tr>
<td>1993</td>
<td>$1112.80</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
</tr>
</tbody>
</table>

c) Plot the rates of return from 1993 to 2007 as a function of the year. Properly label both axes, and give your plot a title.

When you are done, post your m-file to the correct folder:

1. Double-click on “My Network Places”. If it is not on your desktop, look in your start menu.
3. Log in with your email address and password.
4. Double-click on Academic Affairs.
5. Double-click on ME.
6. Double-click on ME123.
7. Double-click on Exams.
8. Double-click on the folder with your section number.
9. Copy and paste your m-file to this folder.

NOTE: All programming must stop at 8:30pm. You will have a few minutes after that to post your file if you need that time.