EE215 - Circuits and Systems
Exam III

Date: April 16, 2014
Name: ________________________________

Open text. Closed notes.
Exam figures are on the last page of the exam.

1. [18%] Consider the periodic function, \( f(t) \), shown in Figure PR-1.

   (a) [6%] What are the average value, \( a_0 \), the period, \( T \), and the fundamental frequency, \( \omega_0 \), of this function?

   \[ a_0 = \]
   \[ T = \]
   \[ \omega_0 = \]

   (b) [6%] What are the first three harmonic frequencies of this function? (The first harmonic is at the fundamental frequency.)

   \[ \omega_1 = \]
   \[ \omega_2 = \]
   \[ \omega_3 = \]

   (c) [6%] Considering your previous answers and the symmetry of the function which of the following is a possible expression for the Fourier Series of \( f(t) \)? (Circle the correct response.)

   i. \[ f(t) = \sum_{n=1}^{\infty} a_n \cos(n\pi t) \]

   ii. \[ f(t) = \sum_{n=1}^{\infty} a_n \cos(n\pi t/2) \]

   iii. \[ f(t) = 1 + \sum_{n=1}^{\infty} a_n \cos(n\pi t/2) \]

   iv. \[ f(t) = \sum_{n=1}^{\infty} a_n \sin(n\pi t) \]
2. [18%] The periodic function, \( f(t) \), shown in Figure PR-2, has the following Fourier Series coefficients:

\[
a_0 = 0 \\
a_n = \begin{cases} 
\frac{-1}{n} & n \text{ even} \\
0 & \text{otherwise}
\end{cases} \\
b_n = \begin{cases} 
\frac{1}{n} & n \text{ odd} \\
0 & \text{otherwise}
\end{cases}
\]

(a) [10%] Sketch the magnitude spectrum of \( f(t) \) below. Include spectrum lines corresponding to the DC and the first four harmonic frequencies. Indicate the location (frequency) and magnitude of each spectral line by writing the corresponding values on the graph.

(b) [8%] The voltage function \( f(t) \) is applied to the input of an ideal bandpass filter. The filter passes through the first two harmonics (the fundamental and the second harmonic) with no change in amplitude or phase of either harmonic. Write the expression for the filter output voltage \( v_0(t) \).

\[
v_0(t) = \]

Richardson

Page 2 of 6

EE215 – Spring 2014
3. [20%] Find the Fourier transforms of the following functions. For full credit write the transforms as a ratio of polynomials (in $j\omega$) when possible.

(a) [5%] $f(t) = e^{-4t}u(t)$

$$F(\omega) =$$

(b) [5%] $f(t) = (t-1)e^{-2t}u(t)$

$$F(\omega) =$$

(c) [5%] $f(t) = 10|t|\cos(20t)$

$$F(\omega) =$$

(d) [5%] $f(t) = \delta(t-2)\frac{1}{t^2-2}$

$$F(\omega) =$$
4. [20%] Find the inverse Fourier transforms of each of the following:

(a) [5%] \( F(\omega) = \frac{10}{(2 + j\omega)^2 + 4} \)
\( f(t) = \)

(b) [5%] \( F(\omega) = \frac{1}{(2 + j\omega)(3 - j\omega)} \)
\( f(t) = \)

(c) [5%] \( F(\omega) = \frac{j\omega}{4 + j\omega} \)
\( f(t) = \)

(d) [5%] \( F(\omega) = \frac{\pi\omega}{\omega - 10} \delta(\omega + 10) + \frac{\pi\omega}{\omega + 10} \delta(\omega - 10) \)
\( f(t) = \)
5. [24%] Consider the circuit shown in Figure PR-5.

(a) [4%] Find the transfer function (or frequency response), \( H(\omega) = \frac{I_o(\omega)}{V_s(\omega)} \), of this circuit. (Note that the output quantity of interest is the circuit current and the transfer function is just the reciprocal of the circuit impedance.)

\[ H(\omega) = \]

(b) [5%] Find the impulse response, \( h(t) \), of this circuit.

\[ h(t) = \]

(c) [5%] Find the output current when \( v_s(t) = 20e^{-10t}u(t) \).

\[ i_o(t) = \]

(d) [5%] Find the output current when \( v_s(t) = 40u(t) \).

\[ i_o(t) = \]

(e) [5%] Find the output current when \( v_s(t) = \delta(t-2) \).

\[ i_o(t) = \]
Exam Figures

Figure PR-1

Figure PR-2

Figure PR-5