1. [18%] The periodic signal, \( x(t) \), shown in Figure PR-1 has the following exponential Fourier series coefficients:

\[
D_n = \frac{j n}{(1 + j n)^2}
\]

(a) [6%] What is the fundamental frequency of \( x(t) \) in both Hertz and rad/s?

\[
f_0 = \quad \text{____________________}
\]

\[
\omega_0 = \quad \text{____________________}
\]

(b) [6%] What is the average value of \( x(t) \)?

\[
\overline{x(t)} = \quad \text{____________________}
\]

(c) [6%] What are the frequency (in Hz), amplitude and phase of the first harmonic?

\[
f_1 = \quad \text{____________________}
\]

\[
C_1 = \quad \text{____________________}
\]

\[
\theta_1 = \quad \text{____________________}
\]
2. [26\%] Find the Fourier transforms of the following signals. For full credit express your result as a ratio of polynomials in the variable $\omega$ (or $j\omega$).

(a) [8\%] $x(t) = [e^{-t} - e^{-3t}]u(t)$

(b) [8\%] $x(t) = e^{-(t-4)}u(t)$

(c) [10\%] $x(t) = e^{-2|t|}\cos(10t)$
3. [26%] Find the inverse Fourier transforms of the following functions:

(a) [8%] \( X(\omega) = \delta(\omega) + \text{sinc}(\omega) \)

(b) [8%] \( X(\omega) = \frac{4}{(j\omega + 1)(j\omega + 2)} \)

(c) [10%] \( X(\omega) = e^{-|\omega|} e^{-j\omega} \)
4. [30%] The high-pass filter shown in Fig PR-4 has a frequency response equal to:

\[ H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{j 2\omega}{1 + j 2\omega} \]

(a) [10%] Find the impulse response, \( h(t) \), of this circuit.

(b) [10%] Find the circuit response \( v_o(t) \) to the input \( v_i(t) = e^{-2t} u(t) \).

(c) [10%] Find the step response \( v_o(t) \), the output in response to \( v_i(t) = u(t) \) of this circuit.

Figure PR-4
### Trigonometric Fourier Series

\[ x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n \omega_0 t) + b_n \sin(n \omega_0 t) \]

\[ a_0 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \, dt \]

\[ a_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \cos(n \omega_0 t) \, dt \]

\[ b_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \sin(n \omega_0 t) \, dt \]

\[ \theta_n = \tan^{-1} \left( \frac{b_n}{a_n} \right) \]

### Compact Fourier Series

\[ x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n \omega_0 t + \theta_n) \]

\[ C_0 = a_0 \]

\[ C_n = \sqrt{a_n^2 + b_n^2} \]

\[ \theta_n = \tan^{-1} \left( \frac{b_n}{a_n} \right) \]

### Exponential Fourier Series

\[ x(t) = \sum_{n=-\infty}^{\infty} D_n e^{j n \omega_0 t} \]

\[ D_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-j n \omega_0 t} \, dt \]

### Conversion Formulas

\[ a_0 = C_0 = D_0 \]

\[ a_n = C_n e^{j \theta_n} \]

\[ b_n = C_n e^{-j \theta_n} \]

\[ a_n = j b_n = C_n e^{j \theta_n} \]

\[ b_n = -j a_n = C_n e^{-j \theta_n} \]

### System Response Formulas

\[ H(\omega) = \frac{|H(\omega)|}{e^{j \phi_0} \cos(\Omega_n)} \]

\[ x(t) = \sum_{n=1}^{\infty} C_n \cos(n \omega_0 t + \theta_n) \]

### Frequency Response

\[ H(\omega) = \left| H(\omega) \right| e^{j \phi_0} \]

### Input

\[ x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n \omega_0 t) + b_n \sin(n \omega_0 t) \]

### Output

\[ y(t) = x(t) * h(t) \]