5.1-4)

a) \( X(z) = \frac{z+1}{z} \)

d) \( X(z) = \frac{1}{4} \frac{z-1}{z^2 - \frac{1}{2} z + \frac{1}{4}} \)

g) \( X(z) = -\frac{z}{(z+1)^2} \)

5.1-5)

a) \( x[n] = [2^n - 3^n] u[n] \)

f) \( x[n] = [3(-1)^n - 3(2)^n + n(2)^{n+1}] u[n] \)

\( x[n] = [(2)^n - (5)^n \cos\left(\frac{\pi}{3} n\right) + (5)^n \sin\left(\frac{\pi}{3} n\right)] u[n] \)

k) \( x[n] = [(2)^n + \sqrt{2}(5)^n \cos\left(\frac{\pi}{3} n - \frac{3\pi}{4}\right)] u[n] \)

5.2-3)

a) \( n^2 u[n] \Leftrightarrow \frac{z(z+1)}{(z-1)^3} \)

b) \( n^2 y^n u[n] \Leftrightarrow \frac{yz(z+y)}{(z-y)^3} \)

c) \( n^3 u[n] \Leftrightarrow \frac{z(z^2 + 4z + 1)}{(z-1)^4} \)

5.3-6)

\( y_0[n] = [1 - (2)^n] 4 u[n] \)

\( y_1[n] = \left[ \frac{1}{2} - 2(2)^n + \frac{3}{2}(3)^n \right] u[n] \)

\( y[n] = \left[ \frac{9}{2} - 6(2)^n + \frac{3}{2}(3)^n \right] u[n] \)

The response of this system was calculated using Octave's \texttt{filter} function. That response was plotted in the graph below as a stem plot. The theoretical response above was plotted on the
same graph as a line plot. There is excellent agreement between the two results, providing verification of the theoretical result.

Here is the Octave file that computes the response and creates the graph:

**PR_05_03_06.m**

5.3-9)

\[ x[n] = \frac{7}{2}(-2)^nu[n] + \frac{1}{2}\delta[n] \]

The output of the system with this input was calculated using Octave. This was plotted as a stem plot. The output expression given in the problem statement was plotted as a line plot. The results agree providing verification that the input expression above is correct.

**Problem 5.3-9**

5.3-11)

a) Two real poles at \( z = 0.5 \) and \( z = 1 \). Not BIBO stable.

b) A ninth order pole at \( z = 0 \). The system is BIBO stable. (All systems with finite duration impulse responses are BIBO stable.)

**PR_05_03_09.m**

5.3-12)

(i) \[ \sum_{k=0}^{n} k = \frac{n(n+1)}{2} u[n] \]
(ii) \[ \sum_{k=0}^{n} k^2 = \frac{n(n+1)(n+1)}{3} u[n] \]

The sums can be treated as the output of a system with transfer function \( H(z) = \frac{z}{z - 1} \) and input (i) \( x[n] = n u[n] \) and (ii) \( x[n] = n^2 u[n] \). The theoretical outputs were plotted (line plots) versus the system outputs as calculated using Octave's filter function in the graph below.

![Graph](PR_05_03_12.m)