Problem 3.49: In the circuit shown in Figure 1, \( R = 250 \, \text{k}\Omega, \ C = 1 \, \mu\text{F} \). The input voltage is \( x(t) = 25 \cos(3 \, t) \, u(t) \) and the initial voltage across the capacitor is \( y(0^-) = 2 \, \text{V} \).

![Figure 1](image1.png)

**Theoretical Solution**
The zero-state response (the response due to the input with all initial conditions set to zero) is equal to
\[
y_{ZSR}(t) = -16e^{-4t}u(t) + 20\cos(3t - 36.87^\circ)u(t)
\]
The zero-input response (the response due to the initial conditions with the input set to zero) is equal to
\[
y_{ZIR}(t) = 2e^{-4t}u(t)
\]
The total response is the sum of these two components
\[
y(t) = -14e^{-4t}u(t) + 20\cos(3t - 36.87^\circ)u(t)
\]

**Verification Using LTSpice**
The circuit was simulated using LTSpice. The LTSpice schematic is shown in Figure 2. Notice that the initial voltage across the capacitor is specified using “IC = 2”. The simulated voltage at node y (the voltage across the capacitor) is displayed in Figure 3. The theoretical result is plotted in the same graph. The two graphs overlap which verifies the theoretical solution.

![Figure 2](image2.png)

![Figure 3](image3.png)
Verification Using Octave

The solution to the corresponding differential equation was found using the **ode45** routine in Octave. The result is shown in Figure 4. The theoretical solution is graphed in the same figure. The two graphs overlap providing additional verification that the theoretical solution is correct. The Octave code used to solve the differential equation and graph the theoretical solution is shown in Listing 1.

![Figure 4](image)

```octave
y0 = 2;
ydot = @(t, y) 100*cos(3*t) - 4*y;
trange = [0:0.05:8];
[t y] = ode45(ydot, trange, y0);
plot(t, y, 'b')
hold on

yt = @(t) -14*exp(-4*t) + 20*cos(3*t - deg2rad(36.87));
plot(t, yt(t), 'r')
hold off
grid on
ylim([-20 20])
xlabel('t (s)')
ylabel('y(t)')
```

Listing 1