Table 2-1: Convolution properties

Convolution Integral $y(t)=h(t)*x(t)=\int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$

- Causal Systems and Signals: $y(t)=h(t)*x(t)=\mathrm{u}(t)\int_{0}^{t} h(\tau)\,x(t-\tau)\,d\tau$

Property	Description
1. Commutative	$x(t)*h(t)=h(t)*x(t)$
2. Associative	$[g(t)*h(t)]*x(t)=g(t)*[h(t)*x(t)]$
3. Distributive	$x(t)*[h_1(t)+\cdots+h_N(t)]=x(t)*h_1(t)+\cdots+x(t)*h_N(t)$
4. Causal*Causal = Causal	$y(t)=\mathrm{u}(t)\int_{0}^{t} h(\tau)\,x(t-\tau)\,d\tau$
5. Time-shift	$h(t-T_1)*x(t-T_2)=y(t-T_1-T_2)$
6. Convolution with Impulse	$x(t)*\delta(t-T)=x(t-T)$
7. Width	Width of $y(t)=$ width of $x(t)+$ width of $h(t)$
8. Area	Area of $y(t)=$ area of $x(t)\times$ area of $h(t)$
9. Convolution with $\mathrm{u}(t)$	$y(t)=x(t)*\mathrm{u}(t)=\int_{-\infty}^{t} x(\tau)\,d\tau$ (Ideal integrator)
10a. Differentiation	$\left(\frac{d^m x}{dt^m}\right)*\left(\frac{d^n h}{dt^n}\right)=\left(\frac{d^{m+n} y}{dt^{m+n}}\right)$
10b. Integration	$\int_{-\infty}^{t} y(\tau)\,d\tau=x(t)*\left[\int_{-\infty}^{t} h(\tau)\,d\tau\right]=\left[\int_{-\infty}^{t} x(\tau)\,d\tau\right]*h(t)$

Table 2.2: Commonly encountered convolutions

No.	$x_1(t)$	$x_2(t)$	$x_1(t) * x_2(t) = x_2(t) * x_1(t)$
1	$u(t)$	$u(t)$	$t\, u(t)$
2	$e^{at} u(t)$	$u(t)$	$\left(\frac{e^{at}-1}{a}\right) u(t)$
3	$e^{at} u(t)$	$e^{bt} u(t)$	$\left[\frac{e^{at}-e^{bt}}{a-b}\right] u(t), \quad a \neq b$
4	$e^{at} u(t)$	$e^{at} u(t)$	$t\, e^{at} u(t)$
5	$t\, e^{at} u(t)$	$e^{bt} u(t)$	$\frac{e^{bt}-e^{at}+(a-b)t\, e^{at}}{(a-b)^2}\, u(t), \quad a \neq b$
6	$t\, e^{at} u(t)$	$e^{at} u(t)$	$\frac{1}{2}\, t^2\, e^{at} u(t)$
7	$\delta(t-T_1)$	$\delta(t-T_2)$	$\delta(t-T_1-T_2)$
8	$t^N\, u(t)$	$e^{at} u(t)$	$\frac{N!\, e^{at}}{a^{N+1}}\, u(t) - \sum_{k=0}^{N} \frac{N!\, t^{N-k}}{a^{k+1}(N-k)!}\, u(t)$
9	$t^M\, u(t)$	$t^N\, u(t)$	$\frac{M!\, N!}{(M+N+1)!}\, t^{M+N+1}\, u(t)$
10	$t^M\, e^{at} u(t)$	$t^N\, e^{at} u(t)$	$\frac{M!\, N!}{(N+M+1)!}\, t^{M+N+1} e^{at}\, u(t)$
11	$t^M\, e^{at} u(t)$	$t^N\, e^{bt} u(t)$	$\sum_{k=0}^{M} \frac{(-1)^k M!\, (N+k)!\, t^{M-k} e^{at}}{k!\, (M-k)!\, (a-b)^{N+k+1}}\, u(t)$ $+\sum_{k=0}^{N} \frac{(-1)^k N!\, (M+k)!\, t^{N-k} e^{bt}}{k!\, (N-k)!\, (b-a)^{M+k+1}}\, u(t)$
12	$e^{-at}\cos(\omega t+\theta)\, u(t)$	$e^{bt} u(t)$	$\frac{\cos(\theta-\phi)e^{bt}-e^{-at}\cos(\omega t+\theta-\phi)}{\sqrt{(a+b)^2+\omega^2}}\, u(t)$ $\phi = \text{atan}2(-\omega,\, a+b)$
13	$\cos(\omega t+\theta)\, u(t)$	$u(t)$	$(1/\beta)[\sin(\omega t+\theta)-\sin(\theta)]\, u(t)$
14	$e^{at} u(t)$	$e^{bt} u(-t)$	$\frac{e^{at}\, u(t)-e^{bt} u(-t)}{b-a} \quad \Re\{b\} > \Re\{a\}$
15	$e^{at} u(-t)$	$e^{bt} u(-t)$	$\frac{e^{at}-e^{bt}}{b-a}\, u(-t)$

Note (assuming $\beta > 0$): $\text{atan}2(-\beta, \alpha+\lambda) = \begin{cases} \tan^{-1}(-\beta/(\alpha+\lambda)) & \text{if } \alpha+\lambda > 0 \\ \tan^{-1}(-\beta/(\alpha+\lambda)) - \pi & \text{if } \alpha+\lambda < 0 \end{cases}$