Table 3-1: Properties of the Laplace transform for causal functions; i.e., $x(t) = 0$ **for** $t < 0$.

	Property	$x(t)$	$\mathbf{X}(\mathbf{s}) = \mathcal{L}[x(t)]$
1	Multiplication by a constant	$K\,x(t)$	$K\,\mathbf{X}(\mathbf{s})$
2	Linearity	$K_1 x_1(t) + K_2 x_2(t)$	$K_1 \mathbf{X}_1(\mathbf{s}) + K_2 \mathbf{X}_2(\mathbf{s})$
3	Time scaling	$x(at)$	$\frac{1}{a}\mathbf{X}\left(\frac{\mathbf{s}}{a}\right), \quad a>0$
4	Time shift	$x(t-T)\,u(t-T)$	$e^{-T\mathbf{s}}\mathbf{X}(\mathbf{s}), \quad T \geq 0$
5	Frequency shift	$e^{-at}x(t)$	$\mathbf{X}(\mathbf{s}+a)$
6	Time 1st derivative	$x' = \frac{dx}{dt}$	$\mathbf{s}\,\mathbf{X}(\mathbf{s}) - x(0^-)$
7	Time 2nd derivative	$x'' = \frac{d^2x}{dt^2}$	$\mathbf{s}^2\mathbf{X}(\mathbf{s}) - \mathbf{s}\,x(0^-) - x'(0^-)$
7a		$x''' = \frac{d^3x}{dt^3}$	$\mathbf{s}^3\mathbf{X}(\mathbf{s}) - \mathbf{s}^2 x(0^-) - \mathbf{s}\,x'(0^-) - x''(0^-)$
7b		$\frac{d^n x}{dt^n}$	$\mathbf{s}^n\mathbf{X}(\mathbf{s}) - \sum_{k=1}^{n} \mathbf{s}^{n-k} x^{(k-1)}(0^-)$
8	Time integral	$\int_{0^-}^{t} x(\tau)\,d\tau$	$\frac{1}{\mathbf{s}}\mathbf{X}(\mathbf{s})$
8a		$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{\mathbf{s}}\mathbf{X}(\mathbf{s}) + \frac{1}{\mathbf{s}}\int_{-\infty}^{0^-} x(\tau)\,d\tau$
9	Frequency derivative	$t\,x(t)$	$-\frac{d}{d\mathbf{s}}\mathbf{X}(\mathbf{s}) = -\mathbf{X}'(\mathbf{s})$
10	Frequency integral	$\frac{x(t)}{t}$	$\int_{\mathbf{s}}^{\infty} \mathbf{X}(\mathbf{z})\,d\mathbf{z}$
11	Initial value	$x(0^+)$	$\lim_{\mathbf{s}\to\infty} \mathbf{s}\,\mathbf{X}(\mathbf{s})$
12	Final value	$\lim_{t\to\infty} x(t) = x(\infty)$	$\lim_{\mathbf{s}\to 0} \mathbf{s}\,\mathbf{X}(\mathbf{s})$
13	Time convolution	$x_1(t) * x_2(t)$	$\mathbf{X}_1(\mathbf{s})\,\mathbf{X}_2(\mathbf{s})$
14	Frequency convolution	$x_1(t)\,x_2(t)$	$\frac{1}{2\pi j}\,\mathbf{X}_1(\mathbf{s}) * \mathbf{X}_2(\mathbf{s})$

Table 3-2: Examples of Laplace transform pairs. Note that $x(t) = 0$ **for** $t < 0^-$ **and** $T \geq 0$.

	$x(t)$	$\mathbf{X}(\mathbf{s}) = \mathcal{L}[x(t)]$
1	$\delta(t)$	1
1a	$\delta(t-T)$	$e^{-T\mathbf{s}}$
2	$u(t)$	$\frac{1}{\mathbf{s}}$
2a	$u(t-T)$	$\frac{e^{-T\mathbf{s}}}{\mathbf{s}}$
3	$e^{-at}u(t)$	$\frac{1}{\mathbf{s}+a}$
3a	$e^{-a(t-T)}u(t-T)$	$\frac{e^{-T\mathbf{s}}}{\mathbf{s}+a}$
4	$t\,u(t)$	$\frac{1}{\mathbf{s}^2}$

4a	$(t-T)u(t-T)$	$\frac{e^{-Ts}}{\mathbf{s}^2}$
5	$t^2u(t)$	$\frac{2}{\mathbf{s}^3}$
5a	$t^nu(t)$	$\frac{n!}{\mathbf{s}^{n+1}}$
6	$te^{-at}u(t)$	$\frac{1}{(\mathbf{s}+a)^2}$
7	$t^2e^{-at}u(t)$	$\frac{2}{(\mathbf{s}+a)^3}$
8	$t^{n-1}e^{-at}u(t)$	$\frac{(n-1)!}{(\mathbf{s}+a)^n}$
9	$\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{\mathbf{s}^2+\omega_0^2}$
9a	$t\ \sin(\omega_0 t)\ u(t)$	$\frac{2\omega_0 s}{(s^2+\omega_0^2)^2}$
10	$\sin(\omega_0 t-\theta)u(t)$	$\frac{-\mathbf{s}\sin\theta+\omega_0\cos\theta}{\mathbf{s}^2+\omega_0^2}$
11	$\cos(\omega_0 t)u(t)$	$\frac{\mathbf{s}}{\mathbf{s}^2+\omega_0^2}$
11a	$t\cos(\omega_0 t)u(t)$	$\frac{\mathbf{s}^2-\omega_0^2}{(\mathbf{s}^2+\omega_0^2)^2}$
12	$\cos(\omega_0 t-\theta)u(t)$	$\frac{\mathbf{s}\cos\theta+\omega_0\sin\theta}{\mathbf{s}^2+\omega_0^2}$
13	$e^{-at}\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$
14	$e^{-at}\cos(\omega_0 t)u(t)$	$\frac{\mathbf{s}+a}{(\mathbf{s}+a)^2+\omega_0^2}$
15	$2e^{-at}\cos(\omega_0 t-\theta)u(t)$	$\frac{e^{j\theta}}{\mathbf{s}+a+j\omega_0}+\frac{e^{-j\theta}}{\mathbf{s}+a-j\omega_0}$
15a	$e^{-at}\cos(\omega_0 t-\theta)u(t)$	$\frac{(\mathbf{s}+a)\cos\theta+\omega_0\sin\theta}{(\mathbf{s}+a)^2+\omega_0^2}$
15b	$re^{-at}\cos(\omega_0 t-\theta)u(t),\quad r=\sqrt{A^2+(B-aA)^2/\omega_0^2},\quad \theta=\tan^{-1}\left(\frac{B-Aa}{A\omega_0}\right)$	$\frac{A\mathbf{s}+B}{(\mathbf{s}+a)^2+\omega_0^2}$
15c	$e^{-at}\left[A\cos\omega_0 t+\frac{B-Aa}{\omega_0}\sin\omega_0 t\right]u(t)$	$\frac{A\mathbf{s}+B}{(\mathbf{s}+a)^2+\omega_0^2}$
16	$\frac{2t^{n-1}}{(n-1)!}e^{-at}\cos(\omega_0 t-\theta)u(t)$	$\frac{e^{j\theta}}{(\mathbf{s}+a+j\omega_0)^n}+\frac{e^{-j\theta}}{(\mathbf{s}+a-j\omega_0)^n}$