In this lab, we will use Fourier series to determine the output of an RC high-pass filter with the input shown in Figure 1.

1. Compute the Fourier Series coefficients $c_k$ for $x(t)$. Do this mathematically (i.e., using calculus, not MATLAB/Octave) using the Fourier series analysis equation

   $$c_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} \, dt$$

   where $T_0$ is the fundamental period of $x(t)$ and $\omega_0 = 2\pi/T_0$. Your derivation of the coefficients $c_k$ should be turned in as part of your lab report.

   For reference, you should obtain as your answer

   $$c_k = \begin{cases} 0 & \text{if } k = 0, \ k \text{ even} \\ -\frac{4}{(\pi k)^2} & \text{if } k \text{ odd} \end{cases}$$

2. Using MATLAB/Octave, plot the magnitude and phase spectrum of $c_k$ for $-10 \leq k \leq 10$. Use the stem command instead of plot to emphasize that the spectra are line spectra and not a continuous function of $k$. Label your axes and title your graphs. Include both graphs in your report.

   Hints: You will need the following MATLAB/Octave commands:
   - abs to compute $|c_k|$.
   - angle to compute the phase angle of $c_k$. (You will need to convert from radians to degrees.)
   - You may wish to write a function to compute $c_k$ for any $k$ - this will be useful later.

3. For the circuit shown in Figure 2 compute the Fourier Series coefficients of the output, $y(t)$, with input $x(t)$.
Include the derivation of $H(s)$ and the output series coefficients, $c_{ky}$, in your lab report. Plot the magnitude and phase spectrum of $c_{ky}$ for $-10 \leq k \leq 10$.

4. Use the MATLAB/Octave \texttt{fs\_exp} function to plot both the circuit input and output using your formulas for the Fourier series coefficients. Use the sum of the first 200 harmonics in the approximation. Plot the functions over the time interval $0 \leq t \leq 5$. Label all axes and title your plots. \textbf{Include both plots in your lab report.}

5. Use LTSpice to simulate the circuit (do a Transient analysis with a Stop Time of 5 s) and find the output when the input is $x(t)$. \textbf{Include your circuit schematic and plots of both the input and output voltage waveforms in your lab report. Compare the your theoretical Fourier series based results to the LTSpice results.} Pay particular attention to differences near $t = 0$ and explain the reason for any differences. (You will need to set the input voltage source to a PULSE source with the following parameters $V_{\text{initial}}= -1$, $V_{\text{on}}=1$, $T_{\text{delay}}=0$, $T_{\text{rise}}=0.5$, $T_{\text{fall}}=0.5$, $T_{\text{on}}=0$, $T_{\text{period}}=1$. Leave the N\texttt{cycles} parameter blank – this corresponds to an infinite number of cycles.)