Bilinear Interpolation Example

Consider enlarging a $M \times M$ image to size $N \times N$ where $M = 500$ and $N = 750$. Imagine scaling the 750 x 750 grid to fit over our 500 x 500 image. Assume our original image is represented by matrix $A$ and the enlarged image by matrix $B$. (The indices of $A$ vary from 0 to 499 while those of $B$ vary from 0 to 749). We need to step over every grid point in the 750 x 750 image (every element in $B$). Consider a single element in matrix $B$, say element $B(332, 615)$. The corresponding scaled coordinates of the grid point are:

\[
\begin{align*}
x &= 332(M-1)/(N-1) = 332(500-1)/(750-1) = 221.185 \\
y &= 615(M-1)/(N-1) = 615(500-1)/(750-1) = 409.726
\end{align*}
\]

Note that these coordinate values will always lie between 0 and 499. The four nearest neighbors of this grid point are: $A(221, 409), A(221, 410), A(222, 409),$ and $A(222, 410)$. (These coordinate values can be found by using the floor and ceil functions.)

The bilinear interpolation equation is:

\[v(x, y) = a \times x + b \times y + c \times x \times y + d\]

We obtain four equations by evaluating this equation at the four nearest neighbors. In matrix form we would have:

\[
\begin{bmatrix}
221 & 409 & 221 \times 409 & 1 \\
221 & 410 & 221 \times 410 & 1 \\
222 & 409 & 222 \times 409 & 1 \\
222 & 410 & 222 \times 410 & 1
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c \\
d
\end{bmatrix} =
\begin{bmatrix}
A(221, 409) \\
A(221, 410) \\
A(222, 409) \\
A(222, 410)
\end{bmatrix}
\]

Assume the four nearest neighbors have intensity values: $A(221, 409) = 121, A(221, 410) = 124, A(222, 409) = 118,$ and $A(222, 410) = 110$. Solving the matrix equation above with these values yields:

\[
\begin{bmatrix}
a & b & c & d
\end{bmatrix} =
\begin{bmatrix}
4496 & 2434 & -11 & -994720
\end{bmatrix}
\]

Using these four coordinates in the bilinear interpolation equation above gives $v(221.185, 409.726) = 121.15$ We would then assign this value to $B(332, 615)$ (after appropriate rounding). This procedure would need to be repeated for each element of $B$.

Notes:

1. The procedure above assumes that the matrix indexes start at 0. If implementing this procedure using Octave/MATLAB you will need to adjust all indices by 1.

2. Special steps may need to be taken at any grid point along the image edges.