Table A6.1 *Fourier-Transform Theorems*

No.	Property	$g(t)$	$G(f) = \mathcal{F}[\,g(t)\,]$
1	Linearity	$a\,g_1(t)+b\,g_2(t)$	$a\,G_1(f)+b\,G_2(f)$ where a and b are constants
2	Dilation (time scaling)	$g(a\,t)$	$\frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$ where a is a constant
3	Duality	$G(t)$	$g(-f)$
4	Time shifting	$g(t-t_0)$	$G(f)\exp(-j\,2\,\pi\,f\,t_0)$
4a	Time translation	$g(a\,t-t_0)$	$\frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)\exp(-j\,2\,\pi\,f\,t_0/a)$
5	Frequency shifting	$\exp(j\,2\,\pi\,f_c\,t)\,g(t)$	$G(f-f_c)$
6	Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)\,dt$	$G(0)$
7	Area under $G(f)$	$g(0)$	$\int_{-\infty}^{\infty} G(f)\,df$
8	Differentiation in time	$\frac{d}{dt}g(t)$	$j\,2\pi\,f\,G(f)$
8a	*n*th derivative in time	$\frac{d^n}{dt^n}g(t)$	$(j\,2\pi\,f)^n G(f)$
9	Integration in time	$\int_{-\infty}^{t} g(\tau)\,d\tau$	$\frac{1}{j\,2\,\pi\,f}G(f)\;+\;\frac{G(0)}{2}\delta(f)$
10	Conjugate functions	$g^*(t)$	$G^*(-f)$
11	Multiplication in time	$g_1(t)\,g_2(t)$	$G_1(f)*G_2(f)=\int_{-\infty}^{\infty} G_1(\lambda)\,G_2(f-\lambda)\,d\lambda$
12	Convolution in time	$g_1(t)*g_2(t)=\int_{-\infty}^{\infty} g_1(\tau)\,g_2(t-\tau)\,d\tau$	$G_1(f)\,G_2(f)$
13	Correlation theorem	$R_{12}(\tau)=\int_{-\infty}^{\infty} g_1(t)\,g_2^*(t-\tau)\,dt$	$G_1(f)\,G_2^*(f)$
14	Rayleigh's theorem	$E=\int_{-\infty}^{\infty} \lvert g(t)\rvert^2\,dt$	$E=\int_{-\infty}^{\infty} \lvert G(f)\rvert^2\,df$
15	Frequency derivative	$t^n\,g(t)$	$\frac{1}{(-j\,2\pi\,f)^n}\frac{d^n}{d\,f^n}G(f)$
16	Conjugate symmetry	$g(t)$ real	$G(-f)=G^*(f)$
17	Time reversal	$g(-t)$	$G(-f)$ or $G^*(f)$
18a	Modulation	$g(t)\cos(2\,\pi\,f_0 t)$	$\frac{1}{2}\left[G(f-f_0)+G(f+f_0)\right]$
18b	Modulation	$g(t)\cos(2\,\pi\,f_0 t+\phi)$	$\frac{1}{2}\left[e^{j\phi}G(f-f_0)+e^{-j\phi}G(f+f_0)\right]$
18c	Modulation	$g(t)\sin(2\,\pi\,f_0 t)$	$\frac{1}{j\,2}\left[G(f-f_0)-G(f+f_0)\right]$
18d	Modulation	$g(t)\sin(2\,\pi\,f_0 t+\phi)$	$\frac{1}{j\,2}\left[e^{j\phi}G(f-f_0)-e^{-j\phi}G(f+f_0)\right]$

Transform: $G(f)=\int_{-\infty}^{\infty} f(t)\,e^{-j\,2\pi\,f\,t}\,dt$ Inverse Transform: $g(t)=\int_{-\infty}^{\infty} G(f)\,e^{j2\pi\,f\,t}\,df$

Table A6.2 *Fourier-Transform Pairs*

No.	Name	$g(t)$	$G(f) = \mathcal{F}[\, g(t) \,]$
1	Pulse of width T	$\mathrm{rect}(t/T)$	$T\ \mathrm{sinc}(f\,T)$
1a	Pulse of width 2T	$\mathrm{rect}(t/(2T))$	$2T\ \mathrm{sinc}(2\,f\,T)$
2	Sinc	$\mathrm{sinc}(2\,W\,t)$	$1/(2\,W)\,\mathrm{rect}(f/(2\,W))$
3	Causal Exponential	$\exp(-a\,t)\,u(t),\quad a>0$	$1/(a+j\,2\,\pi\,f)$
3a	Exp Mult by t^n	$t^n \exp(-a\,t)\,u(t)$	$n!/(a+j\,2\,\pi\,f)^{n+1}$
3b	Anti-Causal Exp	$\exp(a\,t)\,u(-t),\quad a>0$	$1/(a-j\,2\,\pi\,f)$
4	Symmetric Exp	$\exp(-a\lvert t\rvert),\quad a>0$	$2\,a/(a^2+(2\pi\,f)^2)$
5	Gaussian	$\exp(-\pi\,t^2)$	$\exp(-\pi\,f^2)$
5a	Gaussian	$\dfrac{1}{\sqrt{2\pi\sigma^2}}\exp(-t^2/(2\sigma^2))$	$\exp(-\omega^2\sigma^2/2)$
6	Triangular Pulse	$\Delta(t/T)$	$T\,\mathrm{sinc}^2(f\,T)$
7	Impulse	$\delta(t)$	1
8	Unity	1	$\delta(f)$
9	Shifted Impulse	$\delta(t-t_0)$	$\exp(-j\,2\,\pi\,f\,t_0)$
10	Phasor	$\exp(j\,2\,\pi\,f_c t)$	$\delta(f-f_c)$
11	Cosine	$\cos(2\,\pi\,f_c t)$	$(1/2)[\delta(f-f_c)+\delta(f+f_c)]$
12	Sine	$\sin(2\,\pi\,f_c t)$	$(1/(2\,j))[\delta(f-f_c)-\delta(f+f_c)]$
13	Signum (sign)	$\mathrm{sgn}(t)$	$1/(j\,\pi\,f)$
14	Unit Step	$u(t)$	$(1/2)\delta(f)+1/(j\,2\,\pi\,f)$
15	Impulse Train	$\delta_{T_0}(t)=\sum_{i=-\infty}^{\infty}\delta(t-i\,T_0)$	$\dfrac{1}{T_0}\sum_{n=-\infty}^{\infty}\delta\left(f-\dfrac{n}{T_0}\right)$
16a	Causal Exp Sinusoid	$\exp(-a\,t)\sin(2\,\pi\,f_c t)\,u(t)$	$2\,\pi\,f_c/[(a+j\,2\,\pi\,f)^2+(2\pi\,f_c)^2]$
16b	Causal Sinusoid	$\sin(2\,\pi\,f_c t)\,u(t)$	$(1/(4\,j))[\delta(f-f_c)-\delta(f+f_c)]-f_c/[2\,\pi(f_c^2-f^2)]$
17a	Causal Exp Sinusoid	$\exp(-a\,t)\cos(2\,\pi\,f_c t)\,u(t)$	$(a+j\,2\,\pi\,f)/[(a+j\,2\,\pi\,f)^2+(2\pi\,f_c)^2]$
17b	Causal Sinusoid	$\cos(2\,\pi\,f_c t)\,u(t)$	$(1/4)[\delta(f-f_c)+\delta(f+f_c)]+f/[j\,2\,\pi(f^2-f_c^2)]$
18a	Shifted Sine	$\sin(2\,\pi\,f_c t+\phi)$	$(1/(2\,j))[e^{j\phi}\delta(f-f_c)-e^{-j\phi}\delta(f+f_c)]$
18b	Shifted Causal Sine	$\sin(2\,\pi\,f_c t+\phi)\,u(t)$	$(1/(4\,j))[e^{j\phi}\delta(f-f_c)-e^{-j\phi}\delta(f+f_c)]-\dfrac{j\,f\sin(\phi)+f_c\cos(\phi)}{2\,\pi(f^2-f_c^2)}$
19a	Shifted Cosine	$\cos(2\,\pi\,f_c t+\phi)$	$(1/2)[e^{j\phi}\delta(f-f_c)+e^{-j\phi}\delta(f+f_c)]$
19b	Shifted Causal Cosine	$\cos(2\,\pi\,f_c t+\phi)\,u(t)$	$(1/4)[e^{j\phi}\delta(f-f_c)+e^{-j\phi}\delta(f+f_c)]+\dfrac{j\,f\cos(\phi)-f_c\sin(\phi)}{2\,\pi(f^2-f_c^2)}$
20	Absolute value of t	$\lvert t\rvert$	$-2/(2\,\pi\,f)^2$
21	Sinc Squared	$W\,\mathrm{sinc}^2(W\,t)$	$\Delta(f/(2\,W))$

$$\mathrm{rect}(t/T)=u(t+T/2)-u(t-T/2)$$
$$\Delta(t/T)=(1-\lvert t\rvert/T)\,\mathrm{rect}(t/(2\,T))$$
$$\mathrm{sinc}(t)=\sin(\pi\,t)/(\pi\,t)$$